Convex polygon intersection

- The problem: Given two convex polygons, compute their intersection

- Key component in other algorithms, such as
  - computing intersection of half-planes
  - finding the kernel of a polygons
  - linear programming problems
Convex polygon intersection

• Claim: Intersection of two convex polygons P and Q has complexity \(O(|P| + |Q|)\)

• Algorithm outline
  • choose edge A on P, B on Q arbitrarily
  • repeat
    • if A intersects B
      • print intersection (and update inside flag)
      • advance A or B
    • until both A and B cycles their polygons
Advancing

• Idea: the edges A and B chase each other, adjusting so that they meet at each intersection
Advancing

- A directed edge
- H(A): left half-plane of A

point towards A
point away from A
Advancing

• Idea: the edges A and B chase each other, adjusting so that they meet at each intersection

• if both A and B point towards each other
  • advance whichever is outside the other

• if B points towards A and A does not point towards B
  • advance B

• if A points towards B and B does not point towards A
  • advance A

• if neither A and B point towards each other
  • advance whichever is outside the other
A points towards B: advance A
A points towards B: advance A
A points towards B: advance A
A points towards B: advance A
A points away from B, B points away from A: advance whichever is outside the other.
A points away from B, B points away from A: advance whichever is outside the other.
A points towards B: advance A
A points towards B: advance A
A points to B and B towards A: advance B
A points to B, B points to A: advance B
A points to B: advance A
B points to A: advance B

intersection detected