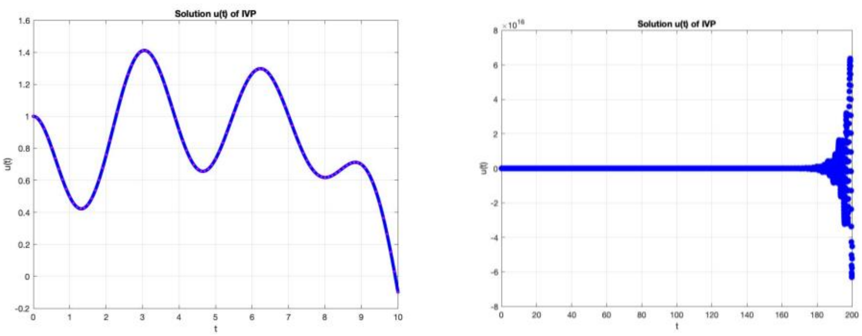
Parametric Instability in Granular Chains
Evelyn Wallace, 2022

For my research project, I considered some hypothetical material in which as vibration passes through, the interactions between the vibration and the material depend on the properties of the material itself. I worked on modeling such a material as a granular chain, which can be represented as a chain of spherical beads each in contact with each adjacent bead. In a traditional or "stable" material, the output vibration closely resembles the input vibration in amplitude and frequency. In an "unstable" material, the output vibration deviates from the input vibration. If the vibration grows larger as it passes through the material, we say that amplification has occurred. Using Newton's second law applied to the simple mass spring model, I was able to model this system of beads to create a system of N ordinary differential equations (ODEs). I used mathematical analysis to solve for the solution of this system working off previous research on modeling granular chains and utilized the programming language Matlab to solve and visualize my model. I considered the variable *A,* that represents the angle of contact between neighboring beads. If the beads are spherical, *A* remains constant (because the rotational symmetry of spheres only allows one discrete angle of contact. I used mathematical analysis and Matlab to model this system of N beads where *A* takes a constant value (k) (Equation 1.A) and confirm that its solution is stable for all parameters as shown in Figure 1A.

$$\ddot{u}_n = k(u_{n-1} - 2u_n + u_{n+1}) \quad \text{for} \quad n = 1, 2, 3, ..., N-1$$ **Equation 1.A**

$$\ddot{u}_n = A(t)(u_{n-1} - 2u_n + u_{n+1}) \quad \text{for} \quad n = 1, 2, 3, ..., N-1$$ **Equation 1.B**

$$\cos(\omega_{1,m} T_\omega) = -\frac{s_1^2 + s_2^2}{2 s_1 s_2} \sin(s_1 \tau) \sin(s_2 (T_\omega - \tau)) + \cos(s_1 \tau) \cos(s_2 (T_\omega - \tau)))$$ **Equation 2.**



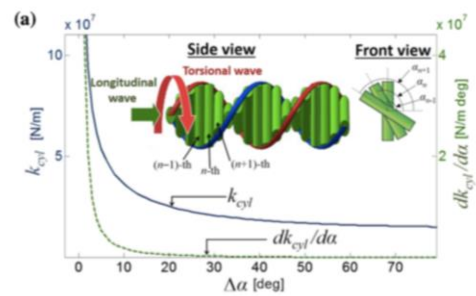


Figure 1. A) Graph of solution to Equation 1.A solved on Matlab does not exhibit amplification (left). **B)** Graph of solution to Equation 1.B solved on Matlab exhibits amplification (middle).

Figure 2. Helicoidal phononic crystals (represented as green cylinders) have a distinct angle of orientation against each neighbor, which determine the contact point between them (right). (F. Li, C. Chong, J. Yang, C. Kevrekidis, and C. Daraio. Time- and Space-variant wave transmission in helicoidal phononic crystals, 2014).

For the material I am interested in studying, I considered beads, such as cylindrical beads, whose angle of contact changes with time. In particular, I considered *A*(t) as a step function (a piecewise function that changes periodically), as shown in Figure 2. I used mathematical analysis and Matlab to model this system of N beads with a step function *A*(t) (Equation 1.B) and confirm that its solution was unstable for certain parameters (as shown in Figure 1.B) and stable for other parameters. I also derived the amplification equation (Equation 2) from this model. Next month, I plan to continue this research as an honors project during the 2021-22 academic year. I will next be exploring the research question: For what parameter values *A*(t) is the granular chain system unstable. I will also be making modifications (such as adding a damping constant) my model to better represent a real-life physical material. Next month I will be working with the Daraio Research Group at CalTech who will run physical experiments based on my simulation of a hypothetical granular chain. I will then use the data obtained from their experiments to test in my simulation.

Faculty Mentor: Christopher Chong
Funded by the Kaufmann Family Fellowship