The Complexity of the Black Scholes Model and the Poisson Process

Renita Shivnauth, 2021

This summer, Renita Shivnauth, under the supervision of Professor Patricia Garmirian, investigated the computational complexity of two well-known processes used to study financial mathematics: the Black Scholes model and the Poisson process. The Black Scholes model is a stochastic, or random, process which models the random movements of stock prices and is based on the well-known Brownian motion process. This model is determined by parameters of the process that may be calculated using finite data sets. The Poisson process, a model also commonly used to study the random movements in financial markets, is characterized by a sequence of jumps at random times. The Poisson model is typically used to model phenomena in the real world in which the representative paths are increasing.

The computational complexity of the processes is analyzed as follows. The paths of each of these processes can be encoded by infinite binary strings (sequences of zeroes and ones). We were interested in the length of the code required to produce a binary string for a given path. If the length of the code grows approximately at the same rate as the length of the binary sequence, then we say that the binary string, and the associated path, are complex in the sense of Kolmogorov Chaitin. Another equivalent definition of complexity is Martin Loff randomness which essentially states that a binary string passes all statistical tests for randomness.

This research project definitively proved that the Black Scholes model was complex both from the point of view of Kolmogorov Chaitin and Martin Loff. This analysis was completed by considering finite “random walk” approximations of Brownian motion and applying a result known as the Mean Value Theorem. We also constructed the Black Scholes model from finite sequences of bits and established a rate and constant of convergence.

In the case of the Poisson process, we defined the complexity by encoding the sequences of jump times. As in the case of the Black Scholes model, we constructed an approximation of the Poisson process which depends on finitely many bits. Further, we proved a relationship between the complexity of a Poisson path and a “modified” complexity of its associated jump times. We also established a correspondence between the space of paths and the space of corresponding binary sequences, and we proved that this correspondence had the “nice” property of being a homeomorphism.