Building a Mathematical Model of the American Lobster Heart

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Mathematical models are extremely useful tools. In the context of biomechanics, math models allow experimenters to toy with the variables of a system free of the constraints of a full experiment. In the Johnson and Dickinson Labs, experiments often center around the function and output of the circulatory system of the American Lobster (*Homarus americanus*) under specific conditions. These conditions vary from stretching the heart by applying force by pulling on the anterior arteries to observing behavior while perfusing the heart with neurotransmitter proteins.

My model is built up from a mechanical analogy between the Windkessel pump and electric circuitry, an analogy that seeks to model the pulse-smoothing behavior of the pump with elementary circuitry (Westerhof *et. al.*, 2008). The 4-Module Winkessel, a unit of circuit that involves a primary resistor in series with a secondary resistor and capacitor in parallel and an inductor, is what I have been using as my unit of biological vessel. In terms of the human circulatory system, the 4-Module Windkessel can be treated as an aorta. In this circuit, Capacitance (C) represents the compliance (stretchiness) of a particular vessel, Resistance (R) represents the vessel resistance, and Inductance (L) represents the blood inertia, which is a measure that recognizes blood as a massive fluid, not an electric current. Voltage (V) and Pressure (P) are analogous, as are Current (I) and Volume Flow Rate (F) (Creigen *et. al.,* 2007). In biological circulatory systems, blood inertia, vessel resistance, and compliance are lumped variable models that all depend on vessel diameter (Ghasemalizadeh *et. al.*, 2008).

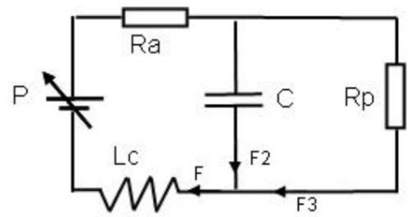

Fig. 1. The 4-Module Windkessel

When analyzing one 4-Module Windkessel, differential definitions of Resistance, Capacitance, and Inductance can be used, resulting in a combined differential equation that represents the behavior of the blood flow and the pressure inside the vessel. However, in the comprehensive circulatory model, I want to string together multiple of these 4-Module Windkessels in series. This requires a different type of circuitry analysis using the concept of complex impedance. From this analysis, I found descriptive equations for both the output pressure and blood volume flow rate of the system. To form a more detailed picture of the system, I looked at the output pressures of each of the individual vessels (recording pressure around the capacitor in each 4-Module Windkessel). From observing the time-varying pressures in each individual vessel, I concluded that to create an effective pulse-smoothing effect, the compliance of the vessels have to become progressively greater relative to the first vessel. As shown in Figure 2, this is consistent with the idea that fluctuations in pressure are much more significant in stiffer (less compliant) vessels than more stretchy (more compliant) vessels. Physiological evidence suggests that vessel walls are more muscular and thus less compliant closer to the heart, and much less muscular and more compliant farther away from the heart. My model also shows that increased resistance in the system translates to increased pressure amplitudes in the peripheral vessels.

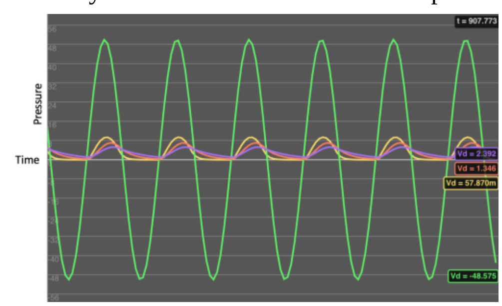

Fig. 2. Pressure vs. Time of sinusoidally driven series of 3 Windkessel circuits. The Windkessel units have ascending compliances: yellow, orange, then purple. The green line is the driving function.

Going forward, I would like to build the pressure and blood flow rate output functions into usable models. Each of these functions for a chain of Windkessels is a complicated string of system variables (resistances, compliances, and inductances), which all uniquely alter the system. I would also like to re-examine the lumped variable models and determine other properties of circulatory systems and vessels, such as discharge time and power output.

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