

Computing Approximate Nash Equilibria in Games

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Game theory studies human behavior in strategic settings. In a game, a set of agents each pick an action from a set of possible actions. A “payoff” of a particular agent depends on the actions of other players in the game. An important concept in game theory is that of Nash equilibrium. In Nash equilibrium, each agent must simultaneously pick an action that maximizes their payoff. This requires no agent could benefit by picking a different action.

It is computationally hard to calculate a Nash equilibrium. Instead, it is common to compute an approximate equilibrium called an ϵ -Nash equilibrium. A common algorithm for computing ϵ -Nash equilibrium is called Nash-Prop, using a scheme called single discretization [1]. This algorithm repeatedly passes messages between agents which describe their best response to their neighbors’ actions. Eventually, these messages converge upon an equilibrium, because agents only share actions that result in a best response from them and their neighbors. The Nash-Prop algorithm works for general games but may not finish within a reasonable amount of time. Another line of research presented an algorithm for computing an approximate Nash equilibrium in a special type of game called graphical multi-hypermatrix games (GMhGs) [2]. This algorithm uses a scheme called double discretization. Although this algorithm is guaranteed to finish quickly, it only works for games where the network connectivity among the agents does not have any cycles.

This summer I developed an algorithm for computing Nash equilibrium in GMhGs with cycles. This algorithm used the double discretization approximation scheme developed in [2] but utilized the same convergence mechanism developed in Nash-Prop to work on networks with cycles. This algorithm works on generalized GMhGs, however the algorithm itself is computationally slow, I am continuing work on the algorithm this year to enable it to converge upon Nash equilibrium faster than is possible using Nash-Prop.

Creating a faster algorithm for computing Nash equilibrium would be a major benefit to the field of game theory. GMhGs have been used to model social influence and animal evolution, and I hope continued work on this algorithm will expand the field and enable us to better model these processes.

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References

- [1] Ortiz, Luis E., and Michael Kearns. "Nash propagation for loopy graphical games." Advances in neural information processing systems 15 (2002).
- [2] Ortiz, Luis E., and Mohammad Irfan. “Tractable Algorithms for Approximate Nash Equilibria in Generalized Graphical Games with Tree Structure.” Proceedings of the AAAI Conference on Artificial Intelligence 31, no. 1 (2017).