Approximation of Discrete Dispersive Shock Waves Through the KdV Equation

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Granular chains are an active topic of research in the mathematical physics community. These chains, effectively a series of a large number of masses connected by springs, can allow for the propagation of dispersive shock waves (DSWs). The waves produced by an initial disturbance in the chain have an amplitude-dependent velocity (i.e. the higher amplitude parts of the wave travel at a different speed than the lower-velocity parts), thereby allowing for the eventual creation of a shock wave that travels through the chain. My work this summer began with familiarising myself with Professor Chong's earlier work on DSWs in granular chains and understanding the differential equation for the strain on the masses in the chain. I and my coworkers worked on writing code to numerically approximate the solution to this equation and reproduce some graphs of this approximation (Figure 1).

The next stage of my work focused on the Korteweg-de Vries (KdV) equation, which can be derived from the original differential equation if some approximation is allowed. I then followed a derivation of an explicit solution to the KdV equation, meaning that I then had a solution into which I could plug values for the mass number and the time to get the strain, as opposed to having the computer computationally approximate the solution all the way up to this mass and time. I then followed another derivation to convert this solution into one which fits with a step function initial condition, that is, an initial strain on the chain equal to a constant non-zero value up to a certain mass and then zero for the remaining masses. I then worked on comparing this explicit solution to the computational solution from our original equation. Producing a graph comparing the two was difficult (the best attempt is Figure 2), given that there are a large number of constants in the two different equations that must be matched in order to have the two solutions describe exactly the same situation. The expected values for these constants have yet to produce a matching graph, so I moved on to running an error analysis of my code in order to identify where the problem was introduced. This error analysis is ongoing, and will hopefully lead to the identification of the error in the code and thus allow the production of a graph matching the two solutions properly.

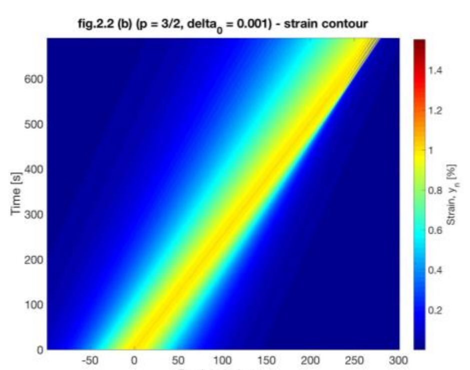

Figure 1

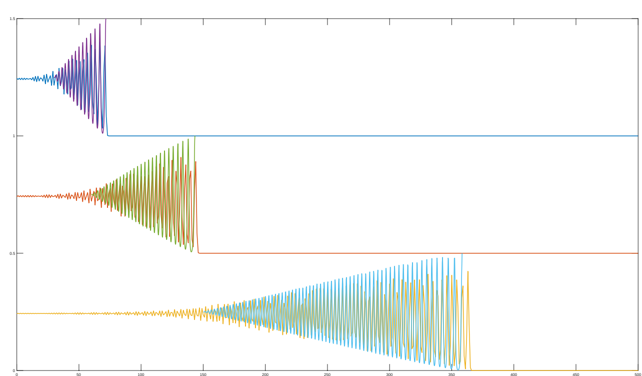

Figure 2

Faculty Mentor: Professor Christopher Chong

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