Resilience in Biological Systems: Analytic Approaches to Flow-kick Dynamical Systems

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In recent years, the scientific community has become increasingly alarmed by current and projected reports of rapid climate change. Global warming, rising sea levels, melting ice caps, and increased frequency of extreme weather events threaten ecosystems which we are reliant on both for resources and for the sustained biodiversity of the planet.[1] In light of these growing concerns, scientists are needing to ask more questions about the resilience of these ecosystems to environmental disturbances. Among these questions are those which address how sustainable management techniques might be used to maintain the desired stable states of ecosystems.

To simulate these scenarios and the ways that management techniques might influence them, we turn to mathematical modeling. Systems of differential equations are well-known models which are useful in understanding the continuous behavior of a natural system; they use series of reasonable, simplifying assumptions to approximate known rules of natural dynamics. We thus use differential equations to model the undisturbed behavior of ecosystems. We must add to this model periodic environmental shocks and management techniques. Helpfully, modeling management patterns is very similar to modeling discrete environmental disturbances; both involve a "kick" in the state variables which represent the ecosystem. The resulting model -- which combines a system of continuous differential equations with discrete, periodic kicks -- is called a *flow-kick dynamical system*.[2]

Flow-kick systems provide a mathematical framework for answering questions such as: *How will this management strategy affect the long term stable state of the system? How resilient would the system be to disturbances in a chosen management strategy?* Techniques for using flow-kick systems to answer these questions have been developed in one dimension.[3] In higher dimensions, however, researchers are currently reliant on computational models and simulations. Though simulations are important and useful, they take time to build and are often computationally expensive.

The aim of this project was to expand analytic understanding of flow-kick systems in higher dimensions. Ultimately, with deeper analytic understanding, comes the development of analytic tools. These tools can inform researches about the expected behavior of a system based on the intrinsic properties of the model. Since they provide insight for researchers without the need of simulations, such analytic tools could reveal which computational analyses would be profitable *ex ante*, thereby increasing the efficiency of the project.

As I worked toward my broad project goal, I refined the focus of my research to the classification of flow-kick equilibria. Previous studies have discussed ways in which continuous dynamical system analysis can be extended to flow-kick systems, but these methods fall short because of the unique nature of flow-kick systems. In contrast to the continuous asymptotic behavior of typical systems, the behavior of flow-kick system depends on the cumulative effects of repeated finite flow periods. The largest portion of my work this summer has focused on using a combination of the variational equation and its reactivity in various regions to characterize this cumulative behavior.[4,5] I intend to extend this work into a year long honors project for the mathematics department.

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