Parametric Instability in an Array of Interacting Magnets

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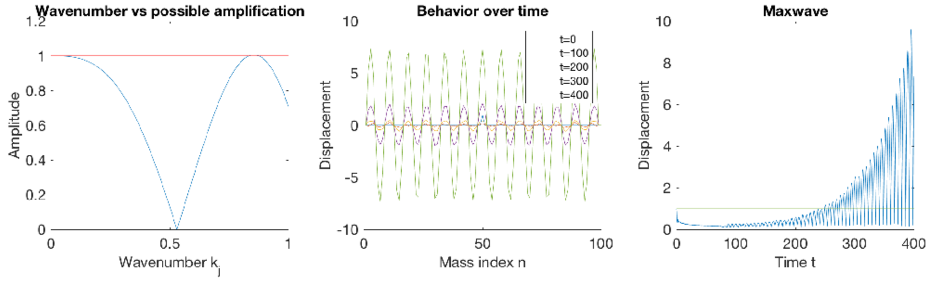
A material with strain-hardening behavior is one that becomes harder to deform as one deforms it; whereas a material with strain-softening behavior becomes *easier* to deform as one deforms the material. There exist many simple examples of strain-hardening materials, one of the most common being a spring, which becomes harder to compress as it is compressed. My research dealt mainly with granular chains with strain-hardening behavior, most simply modeled by a chain of masses connected to each other by springs. Other models exist, for example, an array of interacting magnets. The force exerted by compression or rarefaction of a spring is analogous to the magnetic attraction/repulsion forces between this array of magnets. While we no longer have physical materials exhibiting these behaviors, the forces are therefore still modeled correctly. This magnet experiment setup proves important for the latter sections of my work.

In our mass-spring chain model, this can be shortened to "we assume the mass of any mass in the chain and the spring constant of any spring are constant. The development of the interacting magnet array setup allows us to vary this stiffness while the system is in motion. The goal of this variation is to introduce or avoid instability. Instability can have many impacts, ranging from introducing instability in order to harvest energy, to avoiding it in order to control sound amplitude (acoustic metamaterials) or prevent the collapse of a structure. Thus, this poses the motivation for researching/being able to predict such instability.

This summer, I began by programmatically simulating and understanding the dispersive shockwave in the context of materials exhibiting strain-hardening and strain-softening behavior; specifically in monomer and dimer lattices. In monomer lattices we assume each mass and the springs connecting masses are uniform, whereas in a dimer lattice we have 2 different masses, with every other mass being the same. The code I wrote would numerically model the motion of the masses in the system. I modified the code to adapt to both nonlinear and linear cases to examine amplification in systems with parametric variations in stiffness. For this section, I restricted cases to monomer chains and used a linearized model to derive and analyze a clear, cleaner equation for predicting amplification, one specific manifestation of this instability. The equation, $\cos(\omega T) = -\frac{s_1^2+s_2^2}{2s_1s_2}\sin(s_1\tau)\sin(s_2(T-\tau)) + \cos(s_1\tau)\cos(s_2(T-\tau))$, can directly predict amplification in the linearized model. When the right hand side is greater than 1, no values of w or T will let the left hand side exceed 1. Therefore, when this happens, instability occurs. The graphs below demonstrate a specific simulation where the right hand side of this equation exceeds 1, and show the resulting amplification.



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In conclusion, from the results of my research, we now have a solid understanding of the amplification prediction equation, a derivation of said equation, and programs that allow us to simulate and observe this instability.

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