Obesity Overtaken by Leanness as a Repeated Game:
Social Networks and Indirect Reciprocity

C.F. Runge, E. Nelson, C.P. Runge, J. Levine

Appendix

Note 1

Assume $b_s - c \leq 1$. The societal payoff to both players playing obesity is 2. The societal payoff to both players playing thinness is $2(b_s - c + b_o)$. The societal payoff to one player playing thinness and the other playing obesity is $b_s – c + b_o + 1$. Therefore, as long $b_s - c + b_o > 1$ then the greatest gain to society is form both players playing thinness. Given that $b_s - c \leq 1$, it follows that $b_o > 0$.

Note 2: Equation 2

The first derivative of equation (2) with respect to w_{TFT} is,

$$\left[\left(b_s + b_o\left(n_{TFT|TFT}\right) - c\right)\left(1 + \delta_{TFT}\right)\right] \Big/ \left(1 + \delta_{TFT} - w_{TFT}\right)^2 ,$$

which is always greater than 0 if $n_{TFT|TFT} \geq 2$ and undetermined if $n_{TFT|TFT} < 2$.

The first derivative of equation (2) with respect to δ_{TFT} is,

$$\left[w_{TFT}\left(c - b_s - b_o\left(n_{TFT|TFT}\right)\right)\right] \Big/ \left(1 + \delta_{TFT} - w_{TFT}\right)^2 ,$$

which is always less than 0 if $n_{TFT|TFT} \geq 2$ and undetermined if $n_{TFT|TFT} < 2$.

Note 3: Equation 4

The first derivative of equation (4) with respect to w is,

$$\left(\left(b_o\left(n_{TFT|TFT}\right) + 1\right)\left(1 + \delta_{ABO}\right)\right) \Big/ \left(1 + \delta_{ABO} - w\right)^2 .$$

This derivative is always greater than 0.

Note 4: Equation 6

The first derivative of equation (6) with respect to w is,

$$\left(b_o\left(n_{TFT|TFT}\right) + 1\right)\left(1 + \delta_{ABO}\right) / \left(1 + \delta_{ABO} - w\right)^2,$$

which is always greater than 0.

The first derivative of equation (6) with respect to δ_{ABO} is,

$$-w\left(b_o\left(n_{TFT|TFT}\right)+1\right)/(1+\delta_{ABO}-w)^2,$$

which is always less than 0.

Note 5: Equation 8

The first derivative of equation (8) with respect to w_{ABO} is,

$$\left(b_o\left(n_{TFT|TFT}\right)+1\right)(1+\delta_{ABO})/(1+\delta_{ABO}-w)^2,$$

which is always greater than 0.

The first derivative of equation (8) with respect to δ_{ABO} is,

$$-w_{ABO}\left(b_o\left(n_{TFT|TFT}\right)+1\right)/(1+\delta_{ABO}-w)^2,$$

which is always less than 0.

Note 5A: Inequality 10

The derivative of the right hand side of inequality (10) with respect to w_{TFT},

$$-\frac{(1+\delta_{ABO})(1+\delta_{ABO}-w)(1+c-b_s)}{[w_{TFT}(1+\delta_{ABO})+w(1+\delta_{TFT})]^2},$$

is less than 0. In other words, the right-hand side of inequality (10) decreases in w_{TFT}.

The derivative of the right hand side of inequality (10) with respect to δ_{TFT},

$$\frac{w_{TFT}[(1+\delta_{ABO})^2-w]}{[w_{TFT}(1+\delta_{ABO})+w(1+\delta_{TFT})]^2}$$

is greater than 0. In other words, the right-hand side of inequality (10) increases in δ_{TFT}.

Note 6

Let $w = w_{TFT} = w_{ABO}$. Therefore, inequality (9) becomes,

$$\frac{(b_s-c)(1+\delta_{TFT})+wb_o\left(n_{TFT|TFT}\right)}{1+\delta_{TFT}-w} > \frac{1+\delta_{ABO}+wb_o\left(n_{TFT|TFT}\right)}{1+\delta_{ABO}-w}$$

Solving for $b_s - c$,

$$(b_s - c) > \frac{wb_o\left(n_{TFT|TFT}\right)(\delta_{TFT} - \delta_{ABO}) + (1 + \delta_{ABO})(1 + \delta_{TFT} - w)}{(1 + \delta_{TFT})(1 + \delta_{ABO} - w)}$$

Assume δ_{TFT}= 0.07, δ_{TFT} = 0.10, and w = 0.8 then the above becomes,

$$(b_s - c) > \frac{0.8b_o\left(n_{TFT|TFT}\right)(-0.03) + (1.1)(0.27)}{(1.1)(0.3)}$$

$$(b_s - c) > \frac{-0.024b_o\left(n_{TFT|TFT}\right) + 0.297}{0.33}$$

If $b_o\left(n_{TFT|TFT}\right) = 4$ then the above becomes,

$$(b_s - c) > \frac{-0.096 + 0.297}{0.33} = 0.61$$

Therefore, if δ_{TFT}= 0.07, δ_{TFT} = 0.10, w = 0.8, $b_o\left(n_{TFT|TFT}\right) = 4$, and $(b_s - c) > 0.61$ (which is possible because $b_s - c \leq 1$) then $V(TFT|TFT) > V(ABO|TFT)$.

Let $\delta = \delta_{TFT} = \delta_{ABO}$. Therefore, inequality (9) becomes,

$$\frac{(b_s - c)(1 + \delta) + w_{TFT}b_o\left(n_{TFT|TFT}\right)}{1 + \delta - w_{TFT}} > \frac{1 + \delta + wb_o\left(n_{TFT|TFT}\right)}{1 + \delta - w}$$

Solving for $b_s - c$,

$$(b_s - c) > \frac{1 + \delta + wb_o\left(n_{TFT|TFT}\right) - w_{TFT}\left(1 + b_o\left(n_{TFT|TFT}\right)\right)}{1 + \delta - w}$$

Assume δ = 0.07, w_{TFT} = 0.8, and w = 0.6, then the above becomes,

$$(b_s - c) > \frac{0.27 - 0.2b_o\left(n_{TFT|TFT}\right)}{.47}$$

If $b_o\left(n_{TFT|TFT}\right) = 4$ then the above becomes,

$$(b_s - c) > \frac{0.27 - 0.8}{0.47} = -1.13$$

Therefore, if δ = 0.07, w_{TFT} = 0.8, and w = 0.6, $b_o\left(n_{TFT|TFT}\right) = 4$, and $(b_s - c) > -1.13$ (which is possible because $b_s - c \leq 1$) then $V(TFT|TFT) > V(ABO|TFT)$.