Mathematical Modeling of Underwater Walking in the Crab *Carcinus maenas*

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Summer Research 2009

**Why study underwater walking?**
• Terrestrial walking is well studied, underwater walking is not.
• Our results can be used to improve underwater robots.

**Inverted Pendulum used to Model Walking**
The walker pushes itself up through the first half of the stride, and then is pulled down by gravity during the second half, resulting in a vertical oscillation of the walker’s center of mass. Length of pendulum = leg length; bob of pendulum = walker’s center of mass

*Walking frequency and speed are set by the physical dynamics of the pendulum*

**Terrestrial Froude Number**
• Describes motion of inverted pendulum in air
• Walking is defined to occur at a Fr<sub>air</sub>=0.5, where Fr<sub>air</sub>=<sup>u</sup><sup>2</sup>gl, where u=velocity, g=gravitational acceleration, l=leg length (Alexander, 1989)

**Forces on Underwater Walkers**
Weight (↓); Buoyancy (↑); Lift (↑↓); Drag (acts in opposite direction of motion); Acceleration reaction (force to move added mass) (Martínez, 1996)

**Underwater Froude Number**
• Describes the motion of an inverted pendulum underwater, accounting for buoyancy, added mass, and damping from drag. Used to make predictions about energy costs.

\[ Fr_{\text{water}} = Fr_{\text{air}} \left( \frac{\rho - \rho}{\rho + \kappa \rho} \right) \left( \frac{\beta}{g} \right)^{1/4} \]

\( \rho \) is the object density, \( \rho \) is the fluid density, \( \beta \) is a damping constant, \( \kappa \) is the added mass (Ellers et al., 2009)

• A simpler calculation lowers ‘g’ to account for buoyancy; this is the underwater Fr number (Fr<sub>water</sub>) calculated for this poster (Martínez et al., 1998).

**Goals for the Project**
• Improve models used to describe underwater walking.
• Make predictions about morphologies and densities which are favorable for walking.

**Methods**
1. Collected *C. maenas* of various sizes from the intertidal.
2. Glued white reflective dots onto top of the crab’s carapace as a rough estimate for the crab’s center of mass.
3. Videotaped crabs walking as fast as possible in still water.

**Preliminary Results**
• Fr<sub>water</sub> = 0.14, which is characteristic of a walking gait
• The oscillation of the center of mass seen in many of the crabs also indicates that *C. maenas* can use a walking gait underwater.
• The fit of experimental frequency to that predicted by theory is encouraging.
• Peaks and troughs in crab velocity are likely damped by muscles and tendons.

**Future Work**
• Use other species of crabs (*Hemigrapsus sanguineus*, *Cancer irroratus*).
• Use different underwater walkers (green sea urchin, *Strongylocentrotus droebachiensis*; sea stars *Asterias rubens* and *A. forbesi*).
• Study duty factor to determine the proportion of time one leg spends on the ground during a stride.
• Determine how size, morphology and speed influence gait.

**Literature Cited**

**Acknowledgements**
Thanks to Amy Johnson and Olaf Ellers, Jon Allen and his lab, Alex Brasili, Dan Thornhill and his lab and the Henry L. and Grace Doherty Charitable Foundation and the Hughes Foundation for providing fellowship support.