A.1 Log Linearization

The system consists of equations (5), (13)-(15), (19), (30), and their foreign counterparts, as well as (21)-(23), (27)-(29), and (35)-(38). Some of the equilibrium conditions are log-linear in themselves, such as the money demand equations, the labor supply equations, the risk-sharing condition, and the monetary policy rules.

\[ \hat{M} = \hat{P} + \hat{C} \quad \text{(A.1)} \]
\[ \hat{M}^* = \hat{P}^* + \hat{C}^* \quad \text{(A.2)} \]
\[ \hat{W} = \hat{P} + \hat{C} \quad \text{(A.3)} \]
\[ \hat{W}^* = \hat{P}^* + \hat{C}^* \quad \text{(A.4)} \]
\[ \hat{P} + \hat{C} = \hat{S} + \hat{P}^* + \hat{C}^* \quad \text{(A.5)} \]
\[ \hat{M} = -\delta_s \hat{S} - \delta_p \hat{P}^X \quad \text{(A.6)} \]
\[ \hat{M}^* = -\delta_s^* \hat{S} - \delta_p^* \hat{P}^X \quad \text{(A.7)} \]

First-order approximations of the other equilibrium conditions are given by

\[ \hat{P} = n\hat{P}_H + (1-n)(\hat{P}_F^* + \hat{S}) + O(\epsilon^2) \quad \text{(A.8)} \]
\[ \hat{P}^* = n\hat{P}_H^* + (1-n)\hat{P}_F^* + O(\epsilon^2) \quad \text{(A.9)} \]
\[ \hat{\Lambda} = -\hat{\Lambda} + n\hat{P}_H + (1-n)(\hat{P}_F^* + \hat{S}) + O(\epsilon^2) \quad \text{(A.10)} \]
\[
\hat{\Lambda}^* = -\hat{\Lambda} + n\hat{P}_H^* + (1-n)\hat{P}_F^* + O(\epsilon^2) \quad (A.11)
\]
\[
\hat{P}_H = \hat{P}_H^* = \hat{P}_F^* = \hat{P}_H = \hat{P}_F^* = 0 + O(\epsilon^2) \quad (A.12)
\]
\[
\hat{L} = -\hat{\Lambda} - n\left[\epsilon(\hat{P}_H - \hat{\Lambda}) + (1-\epsilon)\hat{A}\right] - (1-n)\left[\epsilon(\hat{P}_H^* - \hat{\Lambda}^*) + (1-\epsilon)\hat{A}^*\right] + n\hat{C} + (1-n)\hat{C} + O(\epsilon^2) \quad (A.13)
\]
\[
\hat{L}^* = -\hat{\Lambda}^* - n\left[\epsilon(\hat{P}_F^* + \hat{S} - \hat{\Lambda}) + (1-\epsilon)\hat{A}\right] - (1-n)\left[\epsilon(\hat{P}_F^* - \hat{\Lambda}^*) + (1-\epsilon)\hat{A}^*\right] + n\hat{C} + (1-n)\hat{C} + O(\epsilon^2) \quad (A.14)
\]
\[
\hat{Y} = n\left[-\theta(\hat{P}_H - \hat{P}) + \hat{C}\right] + (1-n)\left[-\theta(\hat{P}_H^* - \hat{P}^*) + \hat{C}^*\right] + O(\epsilon^2) \quad (A.15)
\]
\[
\hat{Y}^* = n\left[-\theta(\hat{P}_F - \hat{P}) + \hat{C}\right] + (1-n)\left[-\theta(\hat{P}_F^* - \hat{P}^*) + \hat{C}^*\right] + O(\epsilon^2) \quad (A.16)
\]

**A.2 Figures for Section 4.3**

Figure A.1–Figure A.12 repeat the exercises in the paper by shutting down the final goods sector. In other words, Figure A.1–Figure A.12 correspond to Figure 3, 4, 7, 8, 11, 12, 15, 16, 19, 20, 23, and 24 in the text but with \(\sigma^2_A = \sigma^2_A^* = 0\) and \(\sigma^2_{\tilde{A}} = \sigma^2_{\tilde{A}}^* = 0.0001\).

**A.3 Final Goods PPI Inflation Targeting**

Consider the following inflation targeting regime in which both home and foreign central banks target the PPI of final goods, i.e., \(P^X = \Lambda\), \(P^{*X} = \Lambda^*\), \(\delta_s = \delta_s^* = 0\), and \(\delta_p = \delta_p^* \to \infty\). In this case, exchange rate will respond only to the final goods sector’s productivity shocks and will create inefficient spillovers from the final goods sector to the intermediate goods sector. It first appears that the final goods PPI targeting works just like the intermediate goods PPI targeting where the welfare costs originate from the vertical chain of production and trade. However, these costs are much larger, and indeed unbounded, when it is the intermediate goods sector that faces the negative spillovers. To explain this, let us look at the linearized model.

From equations (A.10)-(A.12), we have
\[
\hat{\Lambda} = -\hat{\Lambda} + (1-n)\hat{S} + O(\epsilon^2) \quad (A.17)
\]
\[ \hat{A}^* = -\hat{A}^* + O(\epsilon^2). \] (A.18)

To replicate the flexible prices of final goods, strict final goods PPI inflation targeting requires

\[
\begin{align*}
\dot{M} &= -\delta_p[-\hat{A} + (1 - n)\hat{S}] + O(\epsilon^2) \\
\dot{M}^* &= -\delta_p^*(-\hat{A}^*) + O(\epsilon^2)
\end{align*}
\] (A.19) (A.20)

where \( \delta_p = \delta_p^* \to \infty \). Moreover, from equations (A.1)-(A.4), we know

\[
\begin{align*}
\hat{W} &= \dot{M} \\
\hat{W}^* &= \dot{M}^*
\end{align*}
\] (A.21) (A.22)

That is to say, wages in each country will respond infinitely to the final goods sector’s productivity shocks, which is inefficient because labor is only used in the intermediate goods sector not in the final goods sector. Such enormous fluctuations in wages, as well as in the unit costs of producing intermediate goods, create huge uncertainties in the economy. Labor supply becomes much more volatile, and intermediate goods producers ask for much higher risk premiums when they preset the prices. As a result, households are expected to consume less, work more, and endure massive volatilities in labor supply. Welfare for both countries are much lower than those under the intermediate goods PPI inflation targeting. In fact, the stricter the final goods PPI inflation targeting (i.e., the higher \( \delta_p \) and \( \delta_p^* \) are), the lower the welfare for each country. As \( \delta_p = \delta_p^* \to \infty \), the welfare for both countries approach negative infinity. This is why we choose not to focus on this type of inflation targeting regime in the paper.
Figure A.1: Home Welfare: LCP+PCP, $\theta = 1$, $\epsilon \in [1, 10]$

Figure A.2: Foreign Welfare: LCP+PCP, $\theta = 1$, $\epsilon \in [1, 10]$

Elasticity of substitution between Home and Foreign Intermediate Goods

Inflation Targeting
Unilateral Peg
Monetary Union
Figure A.3: Home Welfare: LCP+PCP, $\theta = 2$, $\epsilon \in [1,10]$

Figure A.4: Foreign Welfare: LCP+PCP, $\theta = 2$, $\epsilon \in [1,10]$
Figure A.5: Home Welfare: LCP+PCP, $\theta = 5$, $\epsilon \in [1, 10]$

Figure A.6: Foreign Welfare: LCP+PCP, $\theta = 5$, $\epsilon \in [1, 10]$

\[ \varepsilon \text{: Elasticity of substitution between Home and Foreign Intermediate Goods} \]
Figure A.7: Home Welfare: LCP+PCP, $\epsilon = 1$, $\theta \in [1,25]$

Figure A.8: Foreign Welfare: LCP+PCP, $\epsilon = 1$, $\theta \in [1,25]$
Figure A.9: Home Welfare: LCP+PCP, $\epsilon = 2$, $\theta \in [1, 25]$

Figure A.10: Foreign Welfare: LCP+PCP, $\epsilon = 2$, $\theta \in [1, 25]$
Figure A.11: Home Welfare: LCP+PCP, $\epsilon = 5$, $\theta \in [1, 25]$

Figure A.12: Foreign Welfare: LCP+PCP, $\epsilon = 5$, $\theta \in [1, 25]$