Class work: Heaps

1. (CLRS 6.1-1) What are the minimum and maximum number of elements in a heap of height $h$? Note: the height of a heap is the number of edges on the longest root-to-leaf path.

2. (CLRS 6.1-2) Show that an n-element heap has height $\Theta(\lg n)$ (more precisely, $\lceil \lg n \rceil$).

3. (CLRS 6.1-3) Where in a min-heap might the largest element reside, assuming that all elements are distinct?

4. (CLRS 6.1-5) Is an array that is in sorted order a min-heap?

5. (CLRS 6.1-7) Show that the leaves are the nodes indexed by $\lfloor n/2 \rfloor + 1, \ldots, \lfloor n/2 \rfloor + 2, \ldots, n$.

6. What is the effect of calling $\text{HEAPIFY}(A, i)$ for $i > \text{size}[A]/2$? (Sometimes called $\text{HEAPIFY}$ is called DOWN-HEAPIFY, because it heapifies down). Here $i$ is the index of the node where $\text{HEAPIFY}$ is called; initially $i = 1$ (the root).

7. (CLRS 6.5-2) Illustrate the operation of $\text{HEAP-INSERT} (A, 7)$ on the heap (note: this is a min-heap):

   $A = \{2, 5, 10, 6, 8, 100, 11, 9, 15, 9, 10, 200, 101\}$

8. (CLRS 6.5-3) Write pseudocode for the procedure $\text{HEAP-INSERT}$ and $\text{HEAP-DELETE}$ on a min-heap.

9. (CLRS 6.2-1) Illustrate the operation of $\text{HEAPIFY}(A)$ on

   $A = \{(20, 5, 10, 6, 8, 100, 11, 9, 15, 9, 10, 200, 101, 12)\}$

   (note: (Sometimes called $\text{HEAPIFY}$ is called DOWN-HEAPIFY, because it heapifies down.)

10. (CLRS 6.3-1) Illustrate the operation of $\text{BUILD-MAX-HEAP}$ on the array

    $A = \{5, 3, 17, 10, 84, 19, 6, 22, 9\}$

11. (CLRS 6.4-1) Illustrate the operation of Heapsort on the array

    $A = \{5, 13, 2, 25, 7, 17, 20, 8, 4\}$

12. (GT C-2.31) Develop an algorithm that computes the $k$th smallest element in a set of $n$ distinct integers in $O(n + k \lg n)$ time.

13. (CLRS 6.4-3) What is the running time of Heapsort on an array of length $n$ that is already sorted in increasing order? What about decreasing order?