Vertical Trade, Asymmetric Exchange Rate Pass-Through, and Canada's Exchange Rate Regime

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September 2011

Abstract

The recent sharp appreciation of the Canadian dollar vs. the US dollar poses difficulties for the Canadian economy and reignites the debate as to whether Canada should fix the Canadian dollar to the US dollar by implementing a unilateral peg or forming a monetary union. We compare the welfare of different combinations of monetary and currency policies in an open-economy macroeconomic model that incorporates two unique features of the Canadian economy: the high level of vertical trade with the US and the prevalent use of the US dollar as the invoicing currency for both imports and exports. This comparison generates two insights about the exchange rate regime of Canada. First, the vertical trade makes it more difficult for a flexible exchange rate to balance the Canadian economy by adjusting relative prices. The higher degree of economic integration resulting from the vertical production and trade favors a fixed exchange rate regime or a single currency. Second, because Canadian firms often use the US dollar as the invoicing currency for both imports and exports, they undertake most of the currency risk. Relative to other countries with a flexible exchange rate regime, in particular the US, Canada is more exposed to currency fluctuations, which leads to a higher cost of a flexible exchange rate regime in Canada.

JEL classification: F3, F4

Keywords: vertical production and trade, asymmetric exchange-rate pass-through, monetary policy, exchange rate regime

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1 Introduction

The recent sharp appreciation of the Canadian dollar vs. the US dollar poses difficulties for the Canadian economy and reignites the debate as to whether Canada should fix the Canadian dollar to the US dollar by implementing a unilateral peg or forming a monetary union. Indeed, a strong Canadian dollar in a weak economic environment has pushed the Bank of Canada to "talk down" the Canadian dollar: *"A combination of disappointing productivity performance and persistent strength in the Canadian dollar could dampen the expected recovery of Canada's net exports. Heightened tensions in foreign exchange markets could inhibit necessary global adjustment and put additional pressure on freely floating currencies."* (Bank of Canada's Monetary Policy Report, October 2010, p.27).[1]

On the one hand, the Bank is urged to take action in the foreign exchange market to improve the competitiveness of Canadian goods, and hence, to promote economic growth (e.g., Courchene 2007, 2008, Grubel 2008, and Shenfield 2009). On the other hand, the Bank is reminded of its good track record with respect to its monetary policy regarding inflation targeting with a floating exchange rate over the past twenty years and the danger of shifting away from this tradition (e.g., see Bergevin and Busby 2010, Laidler 2008, 2009, Laidler and Robson 2006, Schembri 2006, and Watson 2008). Thus far in the debate, a formal assessment of the cost and benefit of the currency policy is still lacking. In this paper, we provide a welfare comparison of different combinations of monetary and currency policies in an open-economy macroeconomic model featuring prominently the vertical trade and asymmetric exchange rate pass-through.

We believe it is important to take into account three unique features of Canada's international trade when evaluating the effectiveness of Canada's floating exchange rate regime. First, the trading partnership between Canada and the US is the largest in the world, and as such, the majority of Canada's trading is with the US. For example, in 2010, Canada accounted for approximately 19.4% of US exports and approximately 14.5% of US imports.[2] The corresponding numbers are more striking for Canada given that international trade

[1]The recent three issues of the Bank of Canada's Monetary Policy Report (January, April, July 2011) continue to express such concerns.

[2]Source: US Census Bureau Foreign Trade Statistics.

amounts to approximately half of Canada's GDP. In 2010, 50.4% of Canada's imports came from the US, and 74.9% of Canada's exports went to the US.[3]

Second, there is a substantial amount of vertical trade between the two countries, especially in machinery and equipment, industrial goods and materials, and automotive products. While the traditional trade literature has focused on the trade in varieties of (horizontally) differentiated final products (e.g., see Krugman 1979, Lancaster 1980, and Helpman and Krugman 1985), a growing number of recent studies (e.g., see Feenstra 1998, Hummels, Rapoport and Yi 1998, Hummels, Ishii and Yi 2001, Yeats 2001, Yi 2003, and Yi 2010) have emphasized the vertical dimension of the international trade. That is, countries trade not only final goods but also intermediate goods. While the estimates in Hummels et al. (2001) suggest that the vertical specialization share of total exports for Canada was approximately 30% to 35% in 1990, approximately 51% of Canada's export growth is accounted for by the growth in vertical specialization between 1970 and 1990, and Canada's vertical specialization is almost exclusively associated with the developed countries

Finally, empirical evidence suggests that the US dollar is the dominant invoicing currency in the Canada-US trade. Using a novel dataset that covers all Canadian import transactions from 2002 to 2009, Goldberg and Tille (2009) find that over 95% of Canada's imports from the US are invoiced in US dollars by count and over 80% by value. The rest of Canada's imports from the US are mostly priced in Canadian dollars, leaving little room for other currencies. Based on data from the US Bureau of Labor Statistics (International Trade Division), Donnenfeld and Haug (2008) show that, during the period from 1996 to 1998, 92.8% of Canada's exports to the US are invoiced in US dollars, 4.8% in Canadian dollars, and 2.4% in other currencies. Murray, Powell and Lafleur (2003) explore a firm-level survey conducted by the Bank of Canada in 2002 and confirm that the US dollar remains dominant in invoicing Canadian exports.

The increasing level of economic integration between Canada and the US fostered by the increased vertical trade raises question about the ability of a flexible exchange rate to balance the Canadian economy. Moreover, due to the prevalent use of the US dollar in trade, the exchange rate pass-through between the US and Canada is asymmetric. Relative

[3]Source: Statistics Canada, Canadian International Merchandise Trade.

to other countries with a flexible exchange rate regime, in particular the US, Canada is more exposed to currency fluctuations, indicating a higher cost of a flexible exchange rate regime in Canada. This gives rise to the following question: Do the trends of increasing vertical trade and the presence of asymmetric exchange rate pass-through alter the comparison of economic welfare under different monetary systems? In particular, how does a multiple currency system with a flexible exchange rate compare to a currency peg or a common currency system?

To answer these questions, we construct a two-country, open-economy macroeconomic model with vertical trade and an asymmetric exchange rate pass-through. In our model, the two countries differ in size, that is, Canada is approximately one-tenth of the US in terms of GDP. There are two stages of production in each country. The final goods sector in each country produces tradable final goods using domestic and imported intermediate goods, while the intermediate goods are produced from labor. The production of each type of goods in each country faces a stage-specific and country-specific productivity shock. Because the production of tradable final goods requires both domestic and imported intermediate goods, vertical trade is necessary. Motivated by the empirical evidence of the asymmetric exchange rate pass-through, we assume that all goods that cross the Canada-US border are priced in US dollars. To facilitate the comparison of our results with those of the standard new open-economy models, we assume that the financial market is complete and that there exists a production subsidy in each stage of production. Consequently, we abstract from distortions caused by incomplete international risk-sharing and monopolistic competition and focus on a single friction - nominal price rigidity. To complete the model, we specify the monetary policy as a simple money supply rule. We then base the comparison of several exchange rate regimes, such as unilateral peg, monetary union, and inflation targeting with a floating exchange rate, on the welfare of each country.

Our main result is that when vertical trade and asymmetric exchange rate pass-through are present, a fixed exchange rate regime can be desirable. This result contrasts sharply against the typical result in the new open-economy macroeconomics literature that small open economies should favor a flexible exchange rate regime. There are two key reasons behind our result. First, in the presence of vertical trade, there are multiple relative prices that need to be adjusted in response to the underlying shocks in different stages of production. However, there is only one exchange rate, which makes it difficult to balance out the entire economy. Indeed if the exchange rate is flexible, an efficient relative price adjustment at one stage of production can have negative spillover effects on the relative price adjustment at another stage of production via the vertical chain of production and trade. Second, the degree of the exchange rate pass-through plays a crucial role in our model. If a country's exports and imports are both priced in its own currency, that is, there is no exchange rate pass-through, the negative spillovers will not occur because the exchange rate does not affect the relative prices. The country would then favor a flexible exchange rate regime even when there is vertical trade.

Our findings are relevant not only for Canada but also for other countries that share similar characteristics with Canada, such as Mexico, some Central and Eastern European countries, and, to a certain extent, some East Asian countries. Meanwhile, we recognize that there are many economic, social, and political factors that affect the choice of exchange rate regimes. Therefore, the goal of our paper is to study the relationship between vertical trade, the exchange rate pass-through, and the exchange rate regime. Our results, however, should not be viewed as an outright prescription for a fixed exchange rate for these countries.

This paper is closely related to the branch of literature in international macroeconomics that emphasizes the vertical integration of production and trade. Huang and Liu (2001) analyze a closed-economy dynamic stochastic general equilibrium (DSGE) model with sticky prices and multiple stages of production. They find that monetary shocks can generate persistent fluctuations in aggregate output. Their model is extended to study the welfare effect of the inflation on intermediate goods price, cross-country correlation on consumption and on output in a two-country model as well as the welfare impact of a unilateral monetary expansion in an open-economy setup (Huang and Liu, 2005, 2004, 2006). Another article that features vertical production and trade is Shi and Xu (2007), which examines an optimal linear monetary policy rule in an analogous open-economy model. Relative to these papers, our contribution is that we study the policy implication of vertical trade on the desirability of a fixed exchange rate regime or a common currency union.

There is also a voluminous literature on a potential currency union between the US and

Canada. Mundell (1961) has argued that Canada and the US would be better served by a western dollar and an eastern dollar than by a Canadian dollar and a US dollar. This is because the eastern parts of both countries are judged to be more similar economically than the eastern and western halves together of either country; the same is true for the western parts of both countries. Courchene and Harris (1999) also note that the key issue of whether Canada is an "optimum currency area"is based on whether the asymmetric external shocks that affect Canada tend to be north-south or east-west.

Our paper contributes to the literature on the exchange rate regime of Canada by providing a quantitative evaluation of the welfare effects of north-south trade integration under different exchange rate regimes. We abstract from the east-west aspect of the Canadian economy because the west is dominated by the resource sector. Prices for primary commodities determined by the world market are quite flexible, facilitating market forces to achieve efficient adjustment without resorting to currency policies. Therefore, this paper mainly focuses on the impact of vertical trade (in the manufacturing sectors between Canada and the US) on the choice of the exchange rate regime.[4]

This paper proceeds as follows. Section 2 describes the structure of the model. Section 3 presents the method used to compute welfare. Section 4 compares alternative monetary policies for the home and foreign countries and highlights the key driving forces of the results. Section 5 discusses the policy implications. Some conclusions then follow.

2 Model

In our model, the world has two countries, home and foreign. Each country is populated by households that are immobile across borders. We normalize the world population to a measure of one. Home households reside on the interval $[0, n]$, and foreign households reside on the interval $(n, 1]$. Hence, the parameter n is the size of the home country. Households derive utility from aggregate consumption, real money balance, and leisure. In each country, there are two production stages, one for intermediates and one for final goods. As illustrated

[4] In a separate paper, we study a small open economy with two sectors, oil and manufacturing, which faces exogenous world oil price shocks. In that paper, we focus on the income tradeoffs between the two sectors due to fluctuations in exchange rates that are driven by external oil shocks.

in Figure 1, the two countries trade intermediate goods, final goods, and assets, but not labor. For simplicity, we abstract from any dynamics by considering a single period model with uncertainty and with two sub-periods, one before and the other after the realization of productivity shocks.[5] Figure 2 explains the sequence of events within the period. First, households enter the period and trade in a complete set of state-contingent nominal bonds that specify payoffs in all possible future states. Next, monetary authorities announce monetary policy rules. Firms then set prices in advance, taking into account their expected demands, marginal costs, and discount factors. After shocks are realized, production and consumption take place, and the exchange rate is determined.

We describe the detailed structure of the home economy. Unless indicated otherwise, the foreign economy has an identical structure. Where appropriate, variables of the foreign economy are denoted with an asterisk.

2.1 Household

The representative household in the home country, taking prices and wages as given, maximizes the following expected utility:

$$U = \mathbb{E}\left[ln(C) + \chi ln(\frac{M}{P}) - \eta L\right] \tag{1}$$

where C is the aggregate consumption comprised of all home and foreign produced final goods

$$C = \left[n^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{2}$$

The parameter θ ($\theta > 0$) is the elasticity of substitution between home and foreign final goods. The fraction $\frac{M}{P}$ denotes real money balances, and L represents labor supply. The parameters χ and η are positive. There is a continuum of home goods and foreign goods of measure n and $1-n$, respectively. Indices of home and foreign produced goods are defined

[5]Our results will hold in an infinite horizon model because of full risk-sharing and complete price stickiness.

as

$$C_H = \left[(\frac{1}{n})^{\frac{1}{\phi}} \int_0^n C_H(i)^{\frac{\phi-1}{\phi}} di \right]^{\frac{\phi}{\phi-1}} \tag{3}$$

$$C_F = \left[(\frac{1}{1-n})^{\frac{1}{\phi}} \int_n^1 C_F(j)^{\frac{\phi-1}{\phi}} dj \right]^{\frac{\phi}{\phi-1}} \tag{4}$$

where $C_H(i)$ denotes the home final goods of variety i, and $C_F(j)$ is the foreign final goods of variety j. The parameter ϕ ($\phi > 1$) is the elasticity of substitution between varieties of goods within each country. The implied aggregate consumer price index is then

$$P = \left[nP_H^{1-\theta} + (1-n)P_F^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{5}$$

where P_H (P_F) represents the price index for home (foreign) final goods sold in the home country,

$$P_H = \left[\frac{1}{n} \int_0^n P_H(i)^{1-\phi} di \right]^{\frac{1}{1-\phi}} \tag{6}$$

$$P_F = \left[\frac{1}{1-n} \int_n^1 P_F(j)^{1-\phi} dj \right]^{\frac{1}{1-\phi}} \tag{7}$$

Home individual demand for home goods i (foreign goods j) is then given by

$$C_H(i) = \frac{1}{n} [\frac{P_H(i)}{P_H}]^{-\phi} C_H \tag{8}$$

$$C_F(j) = \frac{1}{1-n} [\frac{P_F(j)}{P_F}]^{-\phi} C_F \tag{9}$$

where C_H (C_F) refers to home individual demand for all home (foreign) goods,

$$C_H = n(\frac{P_H}{P})^{-\theta} C \tag{10}$$

$$C_F = (1-n)(\frac{P_F}{P})^{-\theta} C \tag{11}$$

Note that the total demand for home goods is $Y^d = nC_H + (1-n)C_H^*$ and the total demand for foreign goods is $Y^{d*} = nC_F + (1-n)C_F^*$. Moreover, the following identities always hold: $\int_0^n P_H(i)C_H(i)di = P_H C_H$, $\int_n^1 P_F(j)C_F(j)dj = P_F C_F$, and $P_H C_H + P_F C_F = PC$.

Residents of each country can purchase a full set of state-contingent nominal bonds; thus, the household budget constraint is

$$PC + M + \sum_{\xi} q(\xi)B(\xi) = WL + \Pi + B + M_0 + T \tag{12}$$

where $q(\xi)$ and $B(\xi)$ represent the price and the amount of a specific bond that pays one unit of home currency in state ξ, and zero in other states. After shocks are realized, households choose the optimal level of consumption C, money holding M, and labor supply L, while households are financed by labor income WL, payoff from the state-contingent bonds B, profit from all domestic firms Π, the initial money balance M_0, and the government transfer T. We assume that the government repays any seigniorage income through the lump-sum transfer. That is, the government budget constraint is $M_0 = M + T$.

The trade in state-contingent nominal bonds will lead to the following risk-sharing condition

$$\frac{1}{PC} = \Gamma_0 \frac{1}{SP^*C^*} \tag{13}$$

where S is the nominal exchange rate defined as the price of foreign currency in terms of home currency. Γ_0 is state invariant and is determined in the initial market for assets.[6] This paper focuses on the effect of vertical trade and asymmetric exchange rate pass-through on the choice of exchange rate regimes. Hence, we simplify the role of the financial markets by assuming that there is complete international consumption risk-sharing. This assumption is typical in much of the extant new open-economy macroeconomics literature. Furthermore, we assume that the monetary rules are announced after the state-contingent bond markets have closed. We would obtain the same results if we made the reverse assumption because our model is static. In a dynamic model, Γ_0 will become endogenous and depend on policy choices, as shown in Senay and Sutherland (2011). In that case, the timing of asset trade may be crucial for policy analysis, but it is beyond the scope of this paper.

Households' other optimization conditions, namely, the money demand function and the

[6]For detailed derivation of the risk-sharing condition, please see Devereux and Engel (2003) and Chari, Kehoe and McGrattan (2002a).

implicit labor supply schedule, are

$$M = \chi PC \tag{14}$$

$$W = \eta PC \tag{15}$$

As a result, the nominal wage is proportional to money in circulation. In addition, the optimization conditions imply that the exchange rate is determined by the relative money supply between the home and foreign countries, $S = \Gamma_0 \frac{M}{M^*}$.

2.2 Final Goods Stage

Home final goods producer i has access to the following technology

$$Y(i) = A\left[n^{\frac{1}{\epsilon}} X_H(i)^{\frac{\epsilon-1}{\epsilon}} + (1-n)^{\frac{1}{\epsilon}} X_F(i)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{16}$$

where A is the stochastic productivity shock at the final goods stage in the home country with $E(logA) = 0$ and $Var(logA) = \sigma_A^2$. The parameter ϵ ($\epsilon > 0$) is the elasticity of substitution between home and foreign intermediate goods. $X_H(i)$ is a basket of home intermediate goods used in the production of home final goods i. $X_F(i)$ is a basket of foreign intermediate goods used in the production of home final goods i. They are defined as

$$X_H(i) = \left[(\frac{1}{n})^{\frac{1}{\phi}} \int_0^n X_H(i,h)^{\frac{\phi-1}{\phi}} dh\right]^{\frac{\phi}{\phi-1}} \tag{17}$$

$$X_F(i) = \left[(\frac{1}{1-n})^{\frac{1}{\phi}} \int_n^1 X_F(i,f)^{\frac{\phi-1}{\phi}} df\right]^{\frac{\phi}{\phi-1}} \tag{18}$$

where h and f index the varieties of home and foreign intermediate goods. Solving the cost minimization problem of final goods producer i, we obtain the unit cost of producing final goods i

$$\Lambda = \frac{1}{A}\left[n(\tilde{P}_H)^{1-\epsilon} + (1-n)(\tilde{P}_F)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{19}$$

where $\tilde{P}_H$ is the price index of home intermediate goods denominated in home currency, and $\tilde{P}_F$ is the price index of foreign intermediate goods sold in the home country and denominated in home currency.

To characterize the asymmetric exchange rate pass-through observed in the data, we assume that home final goods producers set prices in home currency in the home market, and set prices in foreign currency in the foreign market. Namely, the home final goods producers practice local currency pricing (LCP). In contrast, foreign final goods producers always set prices in the foreign currency in both home and foreign markets. That is, they practice producer currency pricing (PCP). Under these pricing assumptions, we do not expect the law of one price or purchasing power parity (PPP) to hold in general. To abstract from distortions associated with monopolistic competition in both final goods and intermediate goods, we further assume that in each stage of production, there exists a production subsidy, $\gamma = \frac{1}{\phi - 1}$, such that the price markup in each stage equals zero. After introducing such subsidies, the only type of distortion left in the economy is nominal price rigidity.

Given the demand structure and the unit cost of final goods, the home firm i's profit maximization problem is

$$\begin{aligned} \max \mathbb{E}\pi(i) = \mathbb{E}D\{&[(1+\gamma)P_H(i) - \Lambda]n[\frac{P_H(i)}{P_H}]^{-\phi}(\frac{P_H}{P})^{-\theta}C \\ &+[S(1+\gamma)P_H^*(i) - \Lambda](1-n)[\frac{P_H^*(i)}{P_H^*}]^{-\phi}(\frac{P_H^*}{P^*})^{-\theta}C^*\} \end{aligned}$$

where D is the discount factor, $D = \frac{1}{PC}$. The optimal pricing equation of the home final goods i sold in the home market is

$$P_H(i) = \frac{\phi}{(\phi-1)(1+\gamma)} \frac{\mathbb{E}\left[D\Lambda P_H^{\phi}(\frac{P_H}{P})^{-\theta}C\right]}{\mathbb{E}\left[DP_H^{\phi}(\frac{P_H}{P})^{-\theta}C\right]} \tag{20}$$

As all prices are preset, by applying symmetry in varieties of final goods, we obtain the optimal pricing equation for the home final goods sold in the home market

$$P_H = \frac{\phi}{(\phi - 1)(1+\gamma)} \frac{\mathbb{E}(D\Lambda P^{\theta} C)}{\mathbb{E}(D P^{\theta} C)} \tag{21}$$

Similarly, we can derive the optimal pricing equation for the home final goods sold in the foreign market

$$P_H^* = \frac{\phi}{(\phi - 1)(1+\gamma)} \frac{\mathbb{E}[D\Lambda (P^*)^{\theta} C^*]}{\mathbb{E}[DS(P^*)^{\theta} C^*]} \tag{22}$$

Regarding the foreign final goods producers, in addition to assuming they always set prices in their own currency, we assume that they do not price to market, that is, $P_F(j) = SP_F^*(j)$. Given that the foreign country is much larger than the home country, it is reasonable to assume that foreign goods producers may find it too costly (e.g., extra information, menu and operation costs) to set a separate price for the home market. Therefore, we can write the foreign firm j's profit maximization problem as

$$\max \mathbb{E}\pi^*(j) = \mathbb{E}D\left\{[(1+\gamma)P_F^*(j) - \Lambda^*]\left[n(\frac{SP_F^*(j)}{P_F})^{-\phi}(\frac{P_F}{P})^{-\theta}C + (1-n)(\frac{P_F^*(j)}{P_F^*})^{-\phi}(\frac{P_F^*}{P^*})^{-\theta}C^*\right]\right\}$$

where $D^* = \frac{1}{P^* C^*}$. In a symmetric equilibrium, we have

$$P_F^* = \frac{\phi}{(\phi - 1)(1+\gamma)} \frac{\mathbb{E}\left\{D^*\Lambda^*[n(\frac{P}{S})^{\theta}C + (1-n)(P^*)^{\theta}C^*]\right\}}{\mathbb{E}\left\{D^*\left[n(\frac{P}{S})^{\theta}C + (1-n)(P^*)^{\theta}C^*\right]\right\}} \tag{23}$$

2.3 Intermediate Goods Stage

Home intermediate goods producer h has the following technology[7]

$$X_H(h) = \tilde{A}L(h) \tag{24}$$

where $\tilde{A}$ is the productivity shock at the intermediate goods stage in the home country with $E(log\tilde{A}) = 0$ and $Var(log\tilde{A}) = \sigma_{\tilde{A}}^2$. From the cost minimization problems of final goods

[7]We have assumed that no capital is required in either stage of production. Because there is no saving-investment decision in a one-period model, capital would be exogenously given in our simple model. Although the initial level of capital stock may have some impact on welfare, this is not the interest of this paper.

producers, we can obtain the total demands for intermediate goods h of all home final goods producers and all foreign final goods producers, $X_H(h)$ and $X_H^*(h)$, respectively,

$$X_H(h) = [\frac{\tilde{P}_H(h)}{\tilde{P}_H}]^{-\phi}(\frac{\tilde{P}_H}{\Lambda})^{-\epsilon}\frac{1}{A^{1-\epsilon}}\int_0^n Y(i)di \tag{25}$$

$$X_H^*(h) = [\frac{\tilde{P}_H^*(h)}{\tilde{P}_H^*}]^{-\phi}(\frac{\tilde{P}_H^*}{\Lambda^*})^{-\epsilon}\frac{1}{(A^*)^{1-\epsilon}}\int_n^1 Y^*(j)dj \tag{26}$$

where $\tilde{P}_H(h)$ is the home-currency price of intermediate goods h in the home country, $\tilde{P}_H^*(h)$ is the foreign-currency price of the intermediate goods h in the foreign country, and $\int_0^n Y(i)di$ and $\int_n^1 Y^*(j)dj$ represent the total output of home and foreign final goods.

As in the case of home final goods producers, we assume that the home intermediate goods producers price in the currencies of the local markets (i.e., LCP). Note that the unit cost of producing intermediate goods is given by $\frac{W}{\tilde{A}}$. The profit maximization problem for home intermediate goods firm h can be written as

$$\max \mathbb{E}D\{[(1+\gamma)\tilde{P}_H(h) - \frac{W}{\tilde{A}}][\frac{\tilde{P}_H(h)}{\tilde{P}_H}]^{-\phi}(\frac{\tilde{P}_H}{\Lambda})^{-\epsilon}\frac{1}{A^{1-\epsilon}}\int_0^n Y(i)di$$
$$+[S(1+\gamma)\tilde{P}_H^*(h) - \frac{W}{\tilde{A}}][\frac{\tilde{P}_H^*(h)}{\tilde{P}_H^*}]^{-\phi}(\frac{\tilde{P}_H^*}{\Lambda^*})^{-\epsilon}\frac{1}{(A^*)^{1-\epsilon}}\int_n^1 Y^*(j)dj\}$$

Again, by exploiting that all prices are preset and applying symmetry across varieties, we obtain the optimal prices of home intermediates in the home and foreign countries

$$\tilde{P}_H = \frac{\phi}{(\phi-1)(1+\gamma)}\frac{\mathbb{E}\left\{D\frac{W}{\tilde{A}}(\Lambda)^{\epsilon}\frac{1}{A^{1-\epsilon}}\int_0^n Y(i)di\right\}}{\mathbb{E}\left\{D(\Lambda)^{\epsilon}\frac{1}{A^{1-\epsilon}}\int_0^n Y(i)di\right\}} \tag{27}$$

$$\tilde{P}_H^* = \frac{\phi}{(\phi-1)(1+\gamma)}\frac{\mathbb{E}\left\{D\frac{W}{\tilde{A}}(\Lambda^*)^{\epsilon}\frac{1}{(A^*)^{1-\epsilon}}\int_n^1 Y^*(j)dj\right\}}{\mathbb{E}\left\{DS(\Lambda^*)^{\epsilon}\frac{1}{(A^*)^{1-\epsilon}}\int_n^1 Y^*(j)dj\right\}} \tag{28}$$

As in the case of foreign final goods producers, we assume that foreign intermediate goods firms price in the foreign currency for both markets (i.e., PCP) and do not engage in pricing to market. Under these assumptions, we can solve for the price of foreign intermediate goods that applies to both the home and foreign markets

$$\tilde{P}_F^* = \frac{\phi}{(\phi-1)(1+\gamma)} \frac{\mathbb{E}\left\{D^* \frac{W^*}{\tilde{A}^*}\left[(\frac{\Lambda}{S})^\epsilon \frac{1}{A^{1-\epsilon}} \int_0^n Y(i)di + (\Lambda^*)^\epsilon \frac{1}{(A^*)^{1-\epsilon}} \int_n^1 Y^*(j)dj\right]\right\}}{\mathbb{E}\left\{D^*[(\frac{\Lambda}{S})^\epsilon \frac{1}{A^{1-\epsilon}} \int_0^n Y(i)di + (\Lambda^*)^\epsilon \frac{1}{(A^*)^{1-\epsilon}} \int_n^1 Y^*(j)dj]\right\}} \quad (29)$$

2.4 Monetary policy

We model monetary policy with a simple money supply rule in which the money supply is targeted on easily observable variables, such as the nominal exchange rate and/or certain price index. If the money supply rule targets a particular price index, it stabilizes this price index; hence, the rule eliminates the relative price distortion at the presence of nominal price rigidities. Because the exchange rate responds endogenously to home and foreign disturbances in this case, such a rule represents a type of active floating exchange rate regime. More precisely, a producer price index (PPI) targeting rule is defined as a rule that replicates the environment of fully flexible prices. Under a PPI targeting rule, although prices are one hundred percent pre-fixed by producers, they are set at a level that is optimal, even after shocks are realized. That is to say, firms will stick to the prices they have chosen even when they have chances to readjust the prices after the state of the economy is revealed. In our model, a price index targeting rule is equivalent to an inflation targeting rule with a zero inflation rate target.

We specify the home monetary rule as

$$M = \bar{M}\left(\frac{S}{\bar{S}}\right)^{-\delta_s}\left(\frac{P^X}{\bar{P}}\right)^{-\delta_p} \quad (30)$$

where $\bar{S}$ is the target level of the exchange rate, and $\bar{P}$ is the target level of the home price. δ_s and δ_p indicate the degree to which the home monetary authority attempts to control variations in the exchange rate and in the price index, respectively. The variable P^X is the "flexible"price, that is, the price that would prevail when prices are flexible. For instance, if the home monetary authority targets the PPI of intermediate goods, then $P^X = \frac{\phi}{(\phi-1)(1+\gamma)}\frac{W}{\tilde{A}}$. Instead, if the home monetary authority targets the PPI of final goods, then $P^X = \frac{\phi}{(\phi-1)(1+\gamma)}\Lambda$.

It is well discussed in the literature that in a typical complete market open economy without vertical trade, it is optimal to stabilize the PPI inflation rate rather than the CPI inflation rate. In this paper, we look at both the final goods and the intermediate goods PPI inflation targeting because in the presence of vertical trade and production, it is not obvious which PPI should be the target. Our main findings hold under both types of price targeting rules. However, from the welfare point of view, targeting the intermediate goods PPI is much more efficient than targeting the final goods PPI. To save space, we focus on the case of targeting the intermediate goods PPI inflation in the main text, and we detail the case of targeting the final goods PPI inflation in the Appendix.

We will analyze three combinations of monetary regimes that we name from the perspective of the home country. These regimes are (1) the "inflation targeting"regime, in which both home and foreign monetary authorities target the PPI of intermediate goods (i.e., $\delta_s = \delta_s^* = 0$ and $\delta_p = \delta_p^* \to \infty$); (2) the "unilateral peg"regime, in which the home country targets the exchange rate (i.e., $\delta_s \to \infty,\ \delta_p = 0$), while the foreign country targets the PPI of intermediate goods ($\delta_s^* = 0,\ \delta_p^* \to \infty$);[8] (3) a coordinated monetary policy regime, the "monetary union", in which the central bank of the monetary union, acting as a social planner, maximizes the world welfare subject to an inflation targeting rule and the constraint that the exchange rate must be kept constant at all times. To be specific, the common central bank maximizes

$$nU + (1-n)U^* \tag{31}$$

subject to

$$\begin{aligned} & n[PC + M + \sum_{\xi} q(\xi)B(\xi)] + (1-n)[SP^*C^* + SM^* + \sum_{\xi} q(\xi)B^*(\xi)] \\ & = n[WL + \Pi + B + M_0 + T] + (1-n)[SW^*L^* + S\Pi^* + B^* + SM_0^* + ST^*] \end{aligned} \tag{32}$$

$$S \equiv 1 \tag{33}$$

$$M \equiv nM + (1-n)M^* \ = \ \bar{M}\left(\frac{P^X}{\bar{P}}\right)^{-\delta_p} \tag{34}$$

[8] We ignore the reverse of case (2), that is, the foreign country pegs unilaterally to the home currency. This is because the foreign economy is much larger in size, and it would not be optimal for the foreign country to engage in a unilateral peg.

where the common central bank targets a weighted average of all the producer prices of intermediate goods, that is, the price target is $P^X = \frac{\phi}{(\phi-1)(1+\gamma)} \left(\frac{W}{\tilde{A}}\right)^n \left(\frac{W^*}{\tilde{A}^*}\right)^{1-n}$ and the policy parameter on the price target δ_p approaches positive infinity.

2.5 Market Clearing

The final goods market clearing conditions are

$$\int_0^n Y(i)di = \left[n^2 \left(\frac{P_H}{P}\right)^{-\theta} C + n(1-n) \left(\frac{P_H^*}{P^*}\right)^{-\theta} C^*\right] \tag{35}$$

$$\int_n^1 Y^*(j)dj = \left[n(1-n) \left(\frac{P_F}{P}\right)^{-\theta} C + (1-n)^2 \left(\frac{P_F^*}{P^*}\right)^{-\theta} C^*\right] \tag{36}$$

The intermediate goods market clearing conditions are

$$\tilde{A}L = (\frac{\tilde{P}_H}{\Lambda})^{-\epsilon} \frac{1}{A^{1-\epsilon}} \int_0^n Y(i)di + (\frac{\tilde{P}_H^*}{\Lambda^*})^{-\epsilon} \frac{1}{(A^*)^{1-\epsilon}} \int_n^1 Y^*(j)dj \tag{37}$$

$$\tilde{A}^*L^* = (\frac{\tilde{P}_F}{\Lambda})^{-\epsilon} \frac{1}{A^{1-\epsilon}} \int_0^n Y(i)di + (\frac{\tilde{P}_F^*}{\Lambda^*})^{-\epsilon} \frac{1}{(A^*)^{1-\epsilon}} \int_n^1 Y^*(j)dj \tag{38}$$

2.6 Equilibrium

The *equilibrium* comprises a set of prices $\{P, P^*, P_H, P_H^*, P_F^*, \tilde{P}_H, \tilde{P}_H^*, \tilde{P}_F^*, \Lambda, \Lambda^*, W, W^*, S\}$ and a set of quantities $\{C, C^*, L, L^*, M, M^*\}$, which solve a system of equations (5), (13)-(15), (19), (30), and their foreign counterparts as well as (21)-(23), (27)-(29) and (37)-(38), given the productivity shocks at each stage of production, A, A^*, $\tilde{A}$, and $\tilde{A}^*$.

3 Solving the Model

Because it is not possible to derive an exact analytical expression for welfare in this model, and a second-order accurate solution is required for welfare comparison,[9] we solve the model by computing the second-order approximation around the non-stochastic steady state.

[9]The need for a second-order accurate solution in welfare analysis has been well addressed in the literature. For example, see Collard and Juillard (2001), Kim and Kim (2003), and Schmitt-Grohe and Uribe (2004).

The second-order approximation of the welfare measure is given by

$$\tilde{U} = E\left[\hat{C} - \eta\bar{L}\left(\hat{L} + \frac{1}{2}\hat{L}^2\right)\right] + O(\epsilon^3) \tag{39}$$

where $\tilde{U}$ is the deviation of welfare from the non-stochastic equilibrium. $\bar{L}$ is the steady-state values of labor supply, and $\eta\bar{L} = 1$. Hereafter, we use the Jonesian hats to denote the log-deviation of a variable x from its steady-state value $\bar{x}$, that is, $\hat{x} = log(x) - log(\bar{x})$. The term $O(\epsilon^n)$ represents residuals of an equation approximated to an order of $n-1$. Welfare is increasing in the expected level of consumption but is decreasing in the expected level of labor and in the variance of labor.

The second-order approximations of the consumption price index in each country are given by

$$\hat{P} = n\hat{P}_H + (1-n)(\hat{P}_F^* + \hat{S}) + \lambda_P + O(\epsilon^3) \tag{40}$$

$$\hat{P}^* = n\hat{P}_H^* + (1-n)\hat{P}_F^* + \lambda_{P^*} + O(\epsilon^3) \tag{41}$$

where λ_P and λ_{P^*} collect the second-order terms

$$\lambda_P = \frac{n(1-n)(1-\theta)}{2}(\hat{P}_H - \hat{P}_F^* - \hat{S})^2 \tag{42}$$

$$\lambda_{P^*} = \frac{n(1-n)(1-\theta)}{2}(\hat{P}_H^* - \hat{P}_F^*)^2 \tag{43}$$

We define terms of trade as the price of exports in terms of imports. Therefore, $E(\lambda_P)$ $(E(\lambda_{P^*}))$ represents the variance of the final goods terms of trade for the home (foreign) country.

Similarly, the second-order approximations of the unit cost of production of final goods in each country are

$$\hat{\Lambda} = -\hat{A} + n\hat{\tilde{P}}_H + (1-n)(\hat{\tilde{P}}_F^* + \hat{S}) + \lambda_\Lambda + O(\epsilon^3) \tag{44}$$

$$\hat{\Lambda}^* = -\hat{A}^* + n\hat{\tilde{P}}_H^* + (1-n)\hat{\tilde{P}}_F^* + \lambda_{\Lambda^*} + O(\epsilon^3) \tag{45}$$

where λ_Λ and λ_{Λ^*} summarize the second-order terms

$$\lambda_\Lambda = \frac{n(1-n)(1-\epsilon)}{2}(\hat{\tilde{P}}_H - \hat{\tilde{P}}_F^* - \hat{S})^2 \tag{46}$$

$$\lambda_{\Lambda^*} = \frac{n(1-n)(1-\epsilon)}{2}(\hat{\tilde{P}}_H^* - \hat{\tilde{P}}_F^*)^2 \tag{47}$$

and $E(\lambda_\Lambda)$ $(E(\lambda_{\Lambda^*}))$ represents the variance of the intermediate goods terms of trade for the home (foreign) country.

The second-order approximations of the optimal pricing equations are given by

$$\hat{P}_H = E(\hat{\Lambda}) + \lambda_{P_H} + O(\epsilon^3) \tag{48}$$

$$\hat{P}_H^* = E(\hat{\Lambda} - \hat{S}) + \lambda_{P_H^*} + O(\epsilon^3) \tag{49}$$

$$\hat{P}_F^* = E(\hat{\Lambda}^*) + \lambda_{P_F^*} + O(\epsilon^3) \tag{50}$$

$$\hat{\tilde{P}}_H = E(\hat{W} - \hat{\tilde{A}}) + \lambda_{\tilde{P}_H} + O(\epsilon^3) \tag{51}$$

$$\hat{\tilde{P}}_H^* = E(\hat{W} - \hat{\tilde{A}} - \hat{S}) + \lambda_{\tilde{P}_H^*} + O(\epsilon^3) \tag{52}$$

$$\hat{\tilde{P}}_F^* = E(\hat{W}^* - \hat{\tilde{A}}^*) + \lambda_{\tilde{P}_F^*} + O(\epsilon^3) \tag{53}$$

where λ_{P_H}, $\lambda_{P_H^*}$, $\lambda_{P_F^*}$, $\lambda_{\tilde{P}_H}$, $\lambda_{\tilde{P}_H^*}$, and $\lambda_{\tilde{P}_F^*}$ collect the second-order terms and represent the risk premium that firms build into the corresponding pricing decisions

$$\lambda_{P_H} = E\{\frac{1}{2}\hat{\Lambda}^2 + \hat{\Lambda}(\theta-1)\hat{P}\} \tag{54}$$

$$\lambda_{P_H^*} = E\{\frac{1}{2}(\hat{\Lambda} - \hat{S})^2 + (\hat{\Lambda} - \hat{S})(\theta-1)\hat{P}^*\} \tag{55}$$

$$\lambda_{P_F^*} = E\{\frac{1}{2}\hat{\Lambda}^{*2} + \hat{\Lambda}^*[n(\theta-1)(\hat{P} - \hat{S}) + (1-n)(\theta-1)\hat{P}^*]\} \tag{56}$$

$$\begin{aligned}\lambda_{\tilde{P}_H} = {} & E\{\frac{1}{2}(\hat{W} - \hat{\tilde{A}})^2 + (\hat{W} - \hat{\tilde{A}})\{\epsilon\hat{\Lambda} - (1-\epsilon)\hat{A} + n[-\theta\hat{P}_H + (\theta-1)\hat{P}] \\ & +(1-n)[-\theta\hat{P}_H^* + (\theta-1)\hat{P}^* - \hat{S}]\}\}\end{aligned} \tag{57}$$

$$\begin{aligned}\lambda_{\tilde{P}_H^*} = {} & E\{\frac{1}{2}(\hat{W} - \hat{\tilde{A}})^2 - \frac{1}{2}\hat{S}^2 + (\hat{W} - \hat{\tilde{A}} - \hat{S})\{\epsilon\hat{\Lambda}^* - (1-\epsilon)\hat{A}^* \\ & +n[-\theta\hat{S} + (\theta-1)\hat{P}] + (1-n)[(\theta-1)\hat{P}^* - \hat{S}]\}\}\end{aligned} \tag{58}$$

$$\begin{aligned}\lambda_{\tilde{P}_F^*} = {} & E\{\frac{1}{2}(\hat{W}^* - \hat{\tilde{A}}^*)^2 + (\hat{W}^* - \hat{\tilde{A}}^*)\{n[\epsilon(\hat{\Lambda} - \hat{S}) - (1-\epsilon)\hat{A}] + n^2[-\theta\hat{P}_H + \hat{S} + (\theta-1)\hat{P}] \\ & +n(1-n)[-\theta\hat{P}_H^* + (\theta-1)\hat{P}^*] + (1-n)[\epsilon\hat{\Lambda}^* - (1-\epsilon)\hat{A}^*] \\ & +n(1-n)[-\theta\hat{P}_F + \hat{S} + (\theta-1)\hat{P}] + (1-n)^2[-\theta\hat{P}_F^* + (\theta-1)\hat{P}^*]\}\}\end{aligned} \tag{59}$$

Eventually, we can express the approximated welfare function (39) in terms of the second moments of variables in the model and compute the second-order accurate second moments from the first-order solutions for the realized values of endogenous variables.[10] The Appendix documents the complete log-linearized system.

4 Results

To compute the welfare, we set the size of the home country to be one-ninth of the foreign country in terms of size, i.e., $n = 0.1$, to match the sizes of the Canadian economy and the American economy. The standard deviation of all productivity shocks is 1% of their steady-state level, i.e., $\sigma_A^2 = \sigma_{A^*}^2 = \sigma_{\tilde{A}}^2 = \sigma_{\tilde{A}^*}^2 = 0.0001$, and all shocks are independent to each other. We rely on graphs to present our results. Because the elasticity of substitution between home and foreign intermediate goods (ϵ) and the elasticity of substitution between home and foreign final goods (θ) are important for the transmission of productivity shocks across the border, we explore a number of parameter ranges and combinations.

As reviewed by Obstfeld and Rogoff (2000), the literature of international economics offers a wide range of estimations for θ from 1.2 to 21.4. The number often used in the macroeconomics study is between 1 and 2, e.g., Backus, Kehoe and Kydland (1994) and Chari, Kehoe and McGrattan (2002b). Meanwhile, the literature on international trade provides much larger estimates of this parameter - often around 10.[11] A value smaller than unity is seldom used. Hence, we argue the most relevant parameter range is $\epsilon,\ \theta > 1$.

4.1 Home Welfare

Figures 3 to 6 plot the home welfare under three combinations of monetary regimes, i.e., inflation targeting, unilateral peg, and monetary union, against θ in the range of $[0, 25]$ while fixing ϵ at 0.5, 1, 2, and 5, respectively. Similarly, Figures 7 to 10 plot the home

[10]This solution technique has been employed in a series of papers. For instance, see Sutherland (2004) and Senay and Sutherland (2005).

[11]Ruhl (2008) reconciles the difference by suggesting that in the international macroeconomics literature, the smaller values of θ correspond to the responses of quantities to transitory shocks, while the larger estimates in the trade literature often rely on responses of quantities to permanent changes in tariff and trade cost.

welfare under three combinations of monetary regimes against ϵ in the range of $[0, 25]$ while fixing θ at 0.5, 1, 2, and 5, respectively.

4.1.1 Unilateral Peg vs. Inflation Targeting

From Figures 3 to 6, we can see that the home welfare under the unilateral peg is not affected by the elasticity of substitution between home and foreign final goods (θ) as a fixed exchange rate eliminates all the relative price changes when prices are sticky. Moreover, the welfare under the inflation targeting is a decreasing function of θ. This is because targeting the producer price of intermediate goods in both countries allows for efficient relative price adjustments at the intermediate goods stage, but not at the final goods stage. As home and foreign final goods become more substitutable (i.e., θ increases), the cost of lacking an efficient relative price adjustment at the final goods stage rises, making the floating exchange rate regime less attractive.

As indicated in Figure 7 to Figure 10, the home welfare under the unilateral peg is positively related to the elasticity of substitution between home and foreign intermediate goods (ϵ). This is because the pricing risks are always higher for home intermediate goods producers than for foreign intermediate goods producers, which means that foreign intermediate goods are constantly cheaper than home intermediate goods in terms of price levels, though not always lower in terms of relative prices. In the case of a unilateral peg, while foreign intermediate goods prices are optimal as they replicate the flexible prices, home intermediate goods prices are much higher due to a complete absence of relative price adjustment. Given the relative price of intermediate goods (i.e., due to a fixed exchange rate and nominal price rigidity), the higher the substitutability between home and foreign intermediate goods (i.e., ϵ increases), the lower is the expected demand for domestic intermediate goods, which reduces the expected level of labor. In other words, when home and foreign intermediate goods become more substitutable, final goods firms can produce the same amount of final goods using a more effective combination of home and foreign intermediate goods. On average, home households are expected to work less, which explains why the welfare of the unilateral peg is an increasing function of ϵ. This result can also be seen in Figure 3 to Figure 6. There, as ϵ rises, the welfare of the unilateral peg goes up for any given value of

θ.

When considering home welfare under the inflation targeting regime, we find it has an interesting inverse U-shape relationship with ϵ. This inverse U-shape relationship occurs mainly because two opposite effects are at work. The first is the relative price effect. As home and foreign intermediate goods become more substitutable, the benefit of having an effective relative price adjustment at the intermediate goods stage increases, which raises the welfare of the inflation targeting regime. The second is a negative spillover effect associated with the feature of vertical production and trade. As the monetary authority targets PPI of intermediate goods, the exchange rate responds only to productivity shocks at the intermediate goods stage in this case. Consequently, any movement in the exchange rate will cause an inefficient relative price adjustment for the final goods. For instance, suppose there is a positive productivity shock to the production of home intermediate goods. This positive shock leads to the depreciation of the home currency. Although prices are sticky, the depreciation of home currency helps to improve efficiency by lowering the relative price of home intermediate goods. Nevertheless, this depreciation of home currency also makes home final goods very cheap. The world demand for final goods shifts toward those made in the home country, even though there is no real fundamental change in the final goods sector. To meet the additional demand with the same technology, home final goods producers must use more intermediate goods, thus causing an excessive demand for the relatively less expensive intermediate goods - the home intermediate goods. To produce more home intermediate goods, home households must supply more labor, resulting in a decrease in their welfare. The negative spillover effect tends to dominate the relative price effect when the value of ϵ is beyond a certain threshold level. Therefore, as ϵ increases, the welfare of the inflation targeting regime first rises and then drops. Moreover, this inverse U-shaped relationship is robust to the value of θ.

There is a third effect - the price level effect - that generates the inverse U-shape relationship. Under the inflation targeting, there is absolutely no pricing risk when the home intermediate goods producers sell in the home market. Neither is there pricing risk when the foreign intermediate goods producers sell in both home and foreign markets because they practice PCP. Nevertheless, owing to LCP, there are still some pricing risks for the home intermediate goods producers to sell in the foreign market. As a result, home intermediate goods are more expensive than foreign intermediate goods in terms of price levels. Similar to the argument outlined in the paragraph above, given the relative price of intermediate goods, the higher the substitutability between home and foreign intermediate goods, the higher is the welfare of home households, as there is less demand for home intermediate goods, that is, less work for home households. However, this effect has limited influence on welfare under the inflation targeting regime because the exchange rate is flexible, and the relative price of intermediate goods varies with the underlying shocks in the economy.

These results show that, in general, for the home country, the unilateral peg dominates the inflation targeting regime, and this result is robust to the value of θ and ϵ.[12]. We can understand the results by focusing on the role of the nominal exchange rate in adjusting relative prices. In the face of external shocks, efficient production and consumption requires adjustment in multiple relative prices throughout the vertical chain of production and trade. However, with nominal prices of intermediate and final goods pre-fixed, one exchange rate cannot achieve efficient adjustments in multiple relative prices simultaneously. Therefore, when there exist prevalent nominal price rigidities and multiple relative prices, a floating exchange rate regime coupled with inflation targeting has limited ability in stabilizing the entire economy.

4.1.2 Unilateral Peg vs. Monetary Union

There is not much difference in home welfare under the monetary union in comparison to that under the unilateral peg, albeit the unilateral peg often dominates the monetary union. Because the exchange rate is fixed under both regimes, the home welfare under the monetary union is also not affected by the elasticity of substitution between home and foreign final goods (θ), as shown in Figure 3 to Figure 6. Meanwhile, under the monetary union, the common central bank targets a weighted average of home and foreign intermediate goods. Neither country's intermediate goods prices are optimal. Compared to the unilateral peg, pricing risks for the home (foreign) intermediate goods producers are smaller (larger) in the

[12]As shown in Figure 3, it is possible that the inflation targeting regime dominates the unilateral peg when both θ and ϵ are very small. However, those are not likely to be the empirically relevant cases.

currency union. However, the reduction (increase) in the pricing risks for home (foreign) intermediate goods producers is very limited because the home country is much smaller than the foreign country. The common central bank gives a much higher weight to the foreign intermediate goods prices. As a result, home intermediate goods still cost more than their foreign counterparts in absolute terms under the monetary union. For the same reason as discussed in Section 4.1.1, the home welfare under the monetary union is again positively related to the elasticity of substitution between home and foreign intermediate goods (ϵ), as shown in Figure 7 - Figure 10. Basically, given the relative price of intermediate goods (i.e., due to single currency and nominal price rigidity), the higher the substitutability between home and foreign intermediate goods, the lower is the weight for home intermediate goods in the optimal basket of all intermediate goods used to produce final goods. Therefore, home households are expected to work less, which leads to an improvement in their welfare. This result can be found in Figure 3 to Figure 6 as well. As ϵ rises, the welfare of the monetary union goes up for any given value of θ.

As previously discussed, the only difference between the monetary union and the unilateral peg is the risk premium included in the pricing of intermediate goods. From the unilateral peg to the monetary union, the pricing risks for home (foreign) intermediate goods producers fall (rise). Accordingly, home intermediate goods become relatively cheaper than foreign intermediate goods. That is, the positive gap between home and foreign intermediate goods prices declines. As the world demand shifts from the foreign to the home intermediate goods, two contrasting effects are at work. On the one hand, as home income rises, home households consume more. On the other hand, home households have to work more, and this effect becomes stronger as home and foreign intermediate goods become more substitutable. From the home country's point of view, the unilateral peg tends to dominate the monetary union, especially for values of ϵ that are more empirically relevant, that is, $\epsilon > 1$.

Putting the results in Sections 4.1.1 and 4.1.2 together, we conclude that for the home country, the unilateral peg generally ranks higher than both the inflation targeting regime and the monetary union.

4.2 Foreign Welfare

Figures 11 to 18 compare the foreign welfare under the three combinations of monetary regimes, that is, inflation targeting, unilateral peg, and monetary union, for different values of θ and ϵ. For the same reasons discussed in Section 4.1, the foreign welfare under the unilateral peg and the monetary union is not affected by θ, and the welfare under the inflation targeting is negatively related to θ. These relationships between welfare and θ in the foreign country mirror those in the home country. Relative to the home country, the foreign country differs in the relationship between ϵ and the welfare under various monetary regimes. With regard to the ranking of the three policy combinations, the foreign country's preference is exactly the opposite of that in the home country. To be specific, for the foreign country, inflation targeting ranks the highest and a monetary union ranks second.

We already know that, regardless of the combination of regimes that is in place, foreign intermediate goods are always cheaper than home intermediate goods in terms of price levels. Therefore, given the relative price of intermediate goods, the higher the substitutability between home and foreign intermediate goods, the lower the welfare of foreign households, as they have to work more to meet the shift in demand. This echoes our previous findings with regard to the home households. The negative effect of ϵ on foreign welfare (i.e., the price level effect), present in all three regimes, explains why the foreign welfares under the unilateral peg and under the monetary union are decreasing functions of ϵ. It also accounts for the initial drop in the foreign welfare under the inflation targeting, that is, the first part of the U-shape between welfare and ϵ. Meanwhile, because the positive effects of the exchange rate on the relative price adjustments between home and foreign intermediate goods (i.e., the relative price effect) increases with ϵ, as discussed in Section 4.1.1, the foreign welfare under the inflation targeting eventually increases with ϵ, explaining the second part of the U-shape relationship. Notice that there is no negative spillover effect for the foreign country due to LCP.

For the foreign country, the welfare ranking between the unilateral peg and the monetary union depends on the differences in the relative price between the home and the foreign intermediate goods. The bigger the gap between home and foreign intermediate goods prices, the more labor the foreign households must supply. Consequently, the monetary union dominates the unilateral peg in the foreign country as long as ϵ is greater than $\frac{1}{2}$. The elasticity ϵ plays a role because foreign households also expect to consume more through complete risk-sharing as we move from the unilateral peg to the monetary union. However, this effect is very small.

The ranking between the monetary union and the inflation targeting for the foreign country depends on both θ and ϵ. Intuitively, if home and foreign final goods are highly substitutable while the substitutability between home and foreign intermediate goods is poor, that is, high θ but low ϵ, the benefit of having efficient relative price adjustments at the intermediate goods stage is limited. In this case, it is possible for the monetary union to outperform the inflation targeting, as illustrated in Figure 11 to Figure 14. Nevertheless, available empirical evidence (Saito 2004) suggests that the international substitutability among the intermediate goods tends to be much higher than that among the final goods, that is, $\epsilon > \theta > 1$.

Overall, our results suggest that for the foreign country, the inflation targeting regime tends to dominate both the unilateral peg and the monetary union.

4.3 Key Driving Forces

By incorporating two important features observed in the Canadian economy, namely, vertical trade and asymmetric exchange rate pass-through, our paper finds that under these conditions, an economy will generally prefer to peg its currency to the foreign country. This result contrasts sharply with those of the standard models in the new open-economy literature, which often favor a flexible exchange rate regime.

To determine whether one of the two features or both features are required for a unilateral peg to be preferred, we conduct a simple experiment. In this experiment, we reduce the importance of final goods sector by shutting down the productivity shocks to the final goods sector. To be specific, we set $\sigma_A^2 = \sigma_{A^*}^2 = 0$ and $\sigma_{\tilde{A}}^2 = \sigma_{\tilde{A}^*}^2 = 0.0001$. As a result, the only source of distortion left in the economy is the sluggish price adjustment at the intermediate goods stage.

In this setup, with no shocks to the final goods production, standard models in the new open-economy macroeconomics literature would typically prescribe that both countries target the PPI of the intermediate goods. However, even with the shocks to the final goods shut down, we find no qualitative change to our earlier results, that is, the home country still prefers a fixed exchange rate regime, while the foreign country mostly prefers an inflation targeting regime.[13]

The key driver behind our results is the asymmetric exchange rate pass-through. It generates inefficient spillovers from the intermediate goods sector to the final goods sector even when intermediate goods prices are optimal and there are no shocks at the final goods stage to be mitigated. Specifically, similar to the discussion in Section 4.1, a flexible exchange rate responds to the intermediate goods sector's productivity shocks, creating changes in the relative price of final goods for the home households who must buy foreign goods priced in terms of the foreign (producer) currency. Such changes in the relative price of final goods are undesirable because there are no shocks to the final goods sector. Meanwhile, the foreign country is immune from these negative spillovers because home goods are priced in the foreign (local) currency in the foreign market.

We should point out that the vertical trade structure is a necessary condition for obtaining our results, although vertical trade itself is not sufficient. Without the vertical trade structure, shocks to the intermediate goods production affect only that stage of production and can be efficiently countered by price targeting. To draw an analogy, the vertical trade structure creates the bridge for inefficient spillovers, but it is the vehicle of exchange rate that delivers the spillovers. If a country has a limited exchange rate pass-through, then even if the bridge of vertical trade is there, no substantial spillovers will arrive. This is exactly the case for the foreign country. However, for countries that have a significant amount of trade priced in foreign currencies, such as Canada, the spillovers created by asymmetric exchange rate pass-through generate serious economic cost associated with a flexible exchange rate regime.

With this simple experiment, we show that for there to be a real tradeoff between the cost of a flexible exchange rate regime due to negative spillovers in the system of vertical production and trade and the benefit of efficient adjustment in the relative price of

[13]See the Appendix for detailed figures.

intermediate goods, both vertical trade and asymmetric exchange rate pass-through must be present. Under these conditions, it is possible to make a case for the fixed exchange rate regime.

5 Policy Implications

We find that a country would tend to favor a one-sided peg if it is small in terms of size, deeply engages in vertical trade with mostly one major country (or trade bloc), and has to absorb most of the exchange rate risk. A vast country, which, by and large, can be considered as a closed-economy, would favor inflation targeting. Hence, in this environment, the preferred policy arrangement for Canada and the US is the unilateral peg regime in which Canada pegs the US dollar, while the US conducts an inflation targeting monetary policy. Our findings are relevant to pairs of countries (or trade blocs) that share characteristics similar to those of Canada and the US.

Many countries experience rapid growth in trade driven by trade in intermediate goods. The first obvious example is Mexico vs. the US. Using data from Mexican maquiladoras, Hummels et al. (2001) find that the vertical specialization share of total exports for Mexico rises steadily between 1984 and 1997 and reaches 32% in 1997. Moreover, Hanson, Mataloni and Slaughter (2005) point out that multinationals account for over half of the US total exports in 1999, and 93% of exports by US parent firms to their foreign manufacturing affiliates are inputs for further processing. As estimated by Burstein, Kurz and Tesar (2008) based on trade flows between US multinationals and their affiliates and between the US and Mexican maquiladoras, the production-sharing intensity of trade, defined as the ratio of affiliate sales of manufactured goods to the US parent as a share of total manufacturing exports to the US in a country (or region), is approximately 50% for Canada and 25% for Mexico in 2003. When maquiladoras are included, the production-sharing intensity of trade increases dramatically from 25% to 55% for Mexico.[14] In addition, Ekanayake, Veeramacheneni and Moslares (2009) find that the increase in intra-industry trade between

[14] As the authors have explained in the paper, these data provide only an imperfect measure of production sharing. For example, they capture only intra-firm trade and omit arm-length production sharing. However, these data are widely used in the literature of assessing the degree of production sharing.

the US and NAFTA during the period from 1990 to 2007 is almost entirely due to two-way trade in vertical differentiation, and there is evidence that the share of vertical intra-industry trade is relatively higher in the Mexico-US trade than in the Canada-US trade.

Another example is the Central and Eastern European countries (CEEC) vs. the European Union (EU). Kaminski and Ng (2001) study ten CEEC,[15] and find that trade in parts has been the most rapidly growing component of international trade for the CEEC. The value of CEEC total trade turnover in parts grew almost threefold from 1993 to 1997, with both exports and imports increasing at a similar pace. The parts are concentrated in three sectors: motor vehicle parts, office machinery, and telecommunication equipment. In 1998, trade with the European Union (EU) accounts for 81.8% of CEEC total imports of parts and 79.4% of their total exports of parts. Furthermore, the most important trading partner with the CEEC is Germany, which accounts for 39.4% of CEEC total imports of parts and 49.7% of their total exports of parts.

There is also abundant evidence of asymmetric exchange rate pass-through between the U.S. and its trade partners and between the eurozone and its trade partners. As documented by Goldberg and Tille (2008), Kamps (2006), and Ligthart and Werner (2010), the US dollar is the primary invoice currency choice in flows to and from the US, and there is an increasing role of the euro in the EU and its accession countries. In particular, Goldberg and Tille (2008) emphasize a "coalescing"effect where exporters try to minimize the movements of their prices relative to their competitors. For instance, trade flows to the US (eurozone) are predominantly invoiced in dollars (euros) because foreign exporters face competition with the US (eurozone) firms.

Our paper suggests that vertical trade and asymmetric exchange rate pass-through provide reasons for why Mexico and the CEEC may prefer to peg the dollar and the euro, respectively, rather than conduct an independent inflation targeting policy. Out of the ten CEEC, Slovenia joined the eurozone in 2007, Slovakia in 2009, and Estonia in 2011. Latvia and Lithuania have proceeded to the European Exchange Rate Mechanism (ERM II), which means that they have fixed their national currency to the euro and must keep inflation in-

[15] Czech Republic, Estonia, Hungary, Poland, Slovenia, Bulgaria, Romania, Slovakia, Latvia, and Lithuania.

line for at least two years before officially joining the eurozone. Our model outlines the vertical trade channel through which these countries can benefit from a fixed exchange rate.

Having said this, we recognize that trade is just one aspect among many in evaluating a country's currency and monetary policy. Issues such as financial and fiscal stability, credibility of the central bank, and social and political factors should all be taken into account when selecting an optimal monetary regime for a country. The contribution of this paper is to identify the welfare effects of the vertical trade channel under different exchange rate regimes. Namely, if a country is highly integrated with a major trading partner through the vertical chain of production and trade and at the same time takes on most of the exchange rate risk, then it will have a preference for a fixed exchange rate regime. Our results should not be viewed, however, as an outright prescription for a fixed exchange rate when these features are present.

Finally, vertical trade is increasingly becoming a defining characteristic of East Asian countries (e.g., see Ando 2006, Athukorala and Yamashita 2006, and Hiratsuka 2008), and most of East Asia's international trade is invoiced in US dollars (e.g., see McKinnon and Schnabl 2004 and Parsons and Sato 2006). Therefore, our paper provides a potential justification for the prevalence of "soft pegging" in East Asia (except Japan), as documented by Calvo and Reinhart (2002) and McKinnon and Schnabl (2004).

An interesting feature of the East Asian production network is that, in addition to the traditional "two-way" trade, there is an increase in "triangular" trade (e.g., see Gaulier, Lemoine and Unal-Kesenci 2007, Hiratsuka 2008, and Lemoine and Unal-Kesenci 2004). For example, Japan exports intermediate goods (e.g., parts and components) to ASEAN countries and China while importing final goods from them. This is similar to the type of vertical trade studied in this paper. Another observation is that Japan and the new industrialized economies such as South Korea, Taiwan, Singapore, Hong Kong export capital goods and intermediate goods to the less developed countries of the region (i.e., ASEAN countries and China), which then process these goods for exports to the US and Europe. Indeed, this "triangular" type of vertical trade has been observed in Mexico and the CEEC, as there is an increasing reliance on Chinese intermediates in this region (e.g., see Gonzalez and Holmes 2011).

One natural extension of the current paper is to study the exchange rate policy question under the "triangular" trade pattern or even a more sophisticated vertical trade structure. It may still be the case that a fixed exchange rate regime is more favorable than a floating exchange rate regime with inflation targeting due to the negative spillovers throughout the vertical chain of production and trade. In a multi-country setup, an additional interesting question would be whether the country or region should fix to a single currency or to a basket of currencies because the invoicing currency can be different for imports and exports and can vary at different stages of production and trade.[16]

6 Conclusion

The Canadian economy shares a very significant trade relationship with the US. In addition, there are two notable features in this relationship. First, the two countries engage in substantial vertical trade, that is, they trade both intermediate inputs and final goods. Second, the vast majority of the trade between the two countries is invoiced in the US dollar, resulting in an asymmetric pass-through of exchange rates with the Canadian side bearing most of the exchange rate risk in trade. By including these features in an open-economy macroeconomic model, we show that compared to a floating exchange rate regime with inflation targeting, a unilateral peg to the US dollar can improve the economic welfare of Canada. Our results are relevant to other small open economies that share similar characteristics with Canada. However, we should caution that the purpose of the paper is to highlight the important welfare effects of vertical trade and asymmetric exchange rate pass-through under different exchange rate regimes. Our results do not imply that small open economies with these features should unquestionably adopt a fixed exchange rate as the choice of monetary and exchange rate regimes should be based on a thorough consideration of various economic and political factors.

[16] Taking a fixed exchange rate regime as given, Xu (2011) studies the optimal weights on "import currency" and "export currency" in a currency basket for a small open economy under the "triangular" vertical trade. An analysis of an optimal exchange rate regime would be interesting in such an environment.

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Figure 1: Structure of the Model

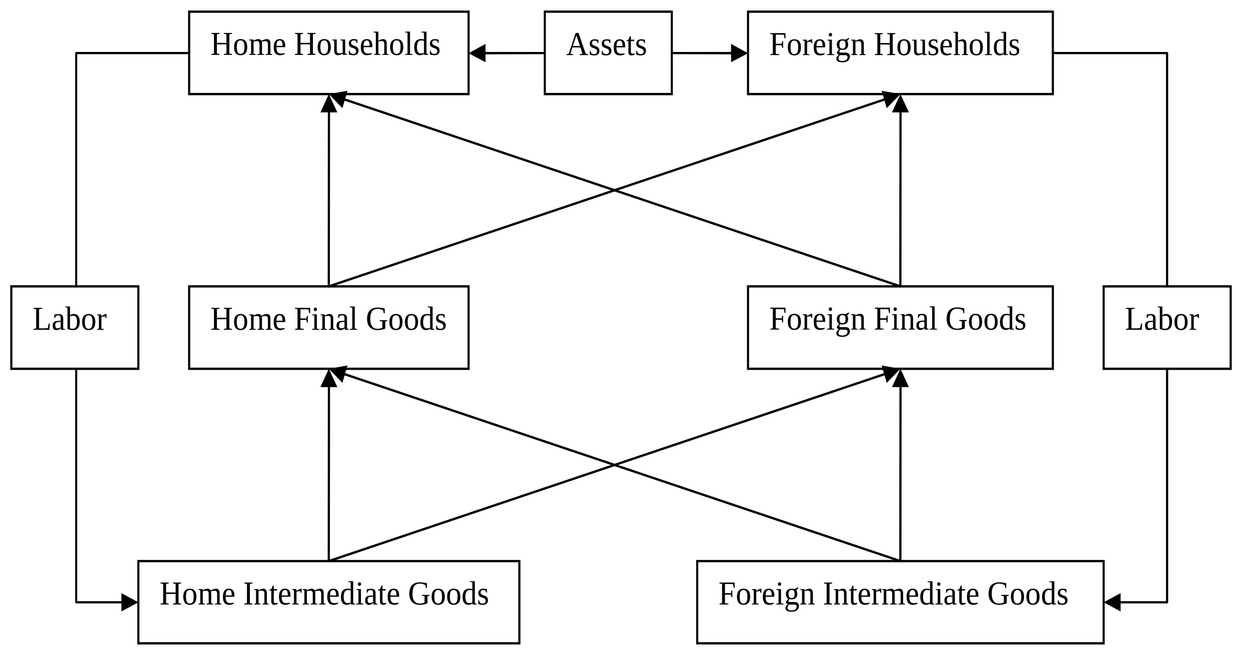


Figure 2: Timing of the Model

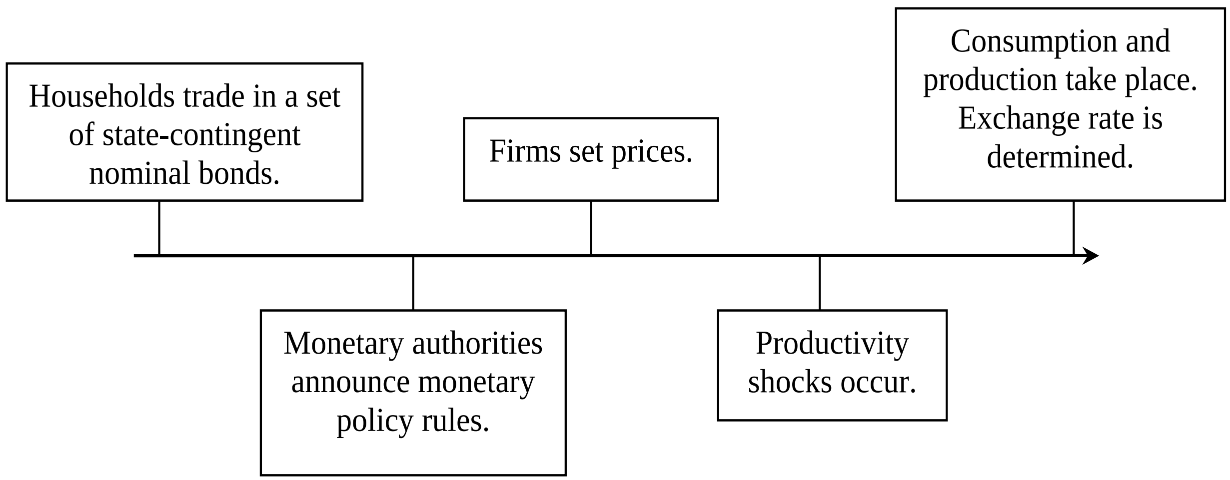


Figure 3: Home Welfare Comparison: $\epsilon = 0.5$, $\theta \in [0, 25]$

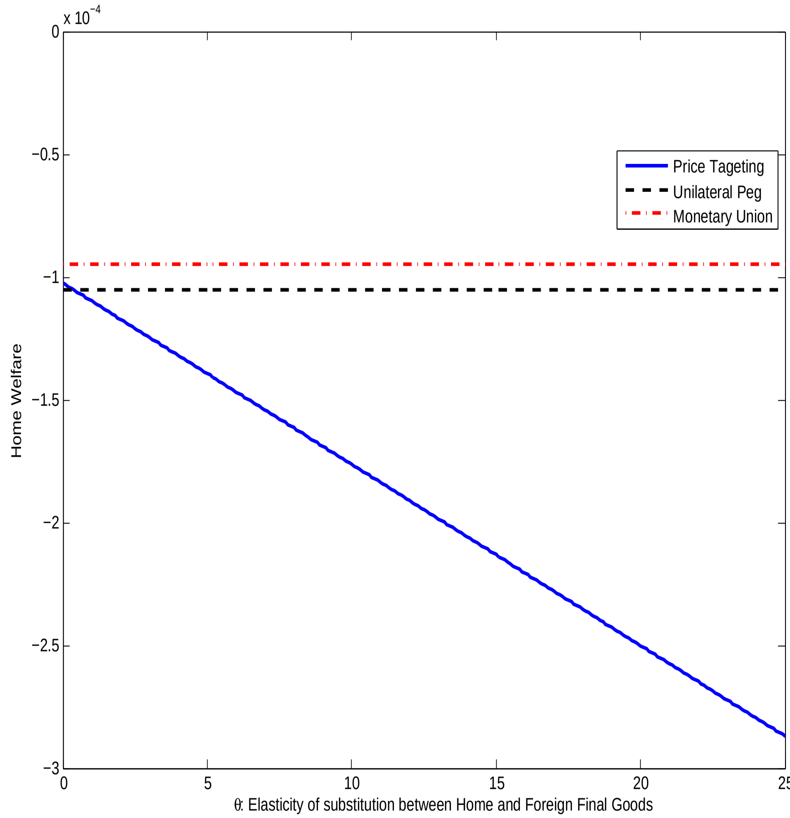


Figure 4: Home Welfare Comparison: $\epsilon = 1$, $\theta \in [0, 25]$

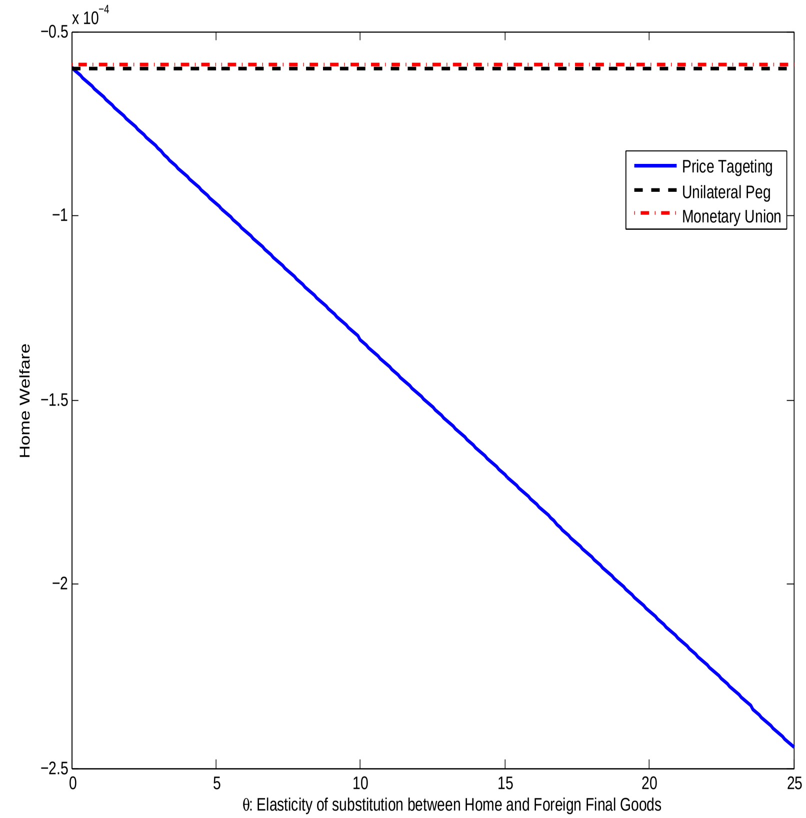


Figure 5: Home Welfare Comparison: $\epsilon = 2$, $\theta \in [0, 25]$

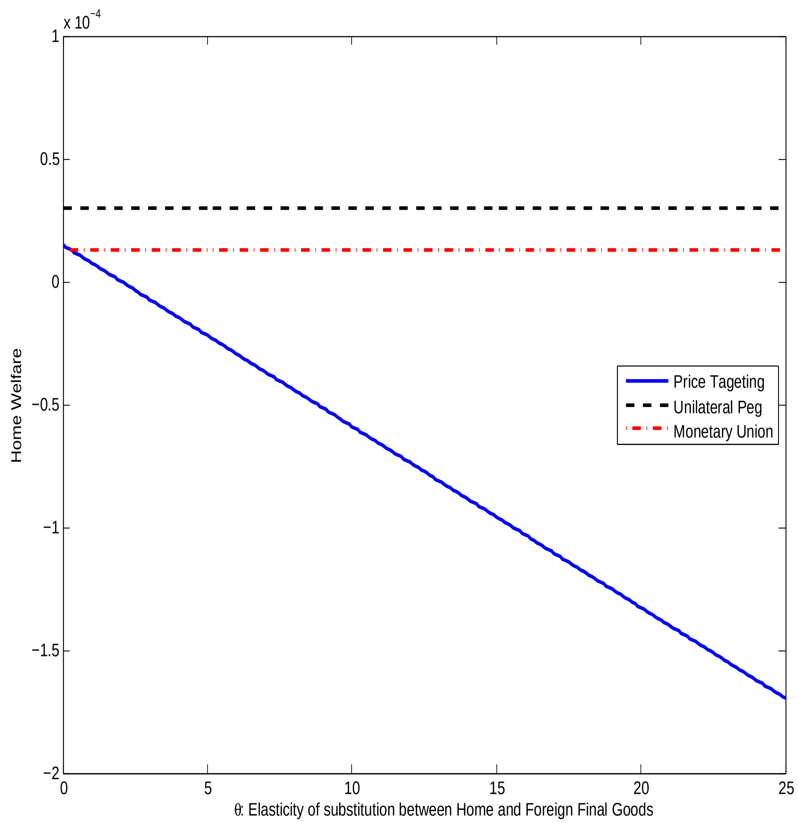


Figure 6: Home Welfare Comparison: $\epsilon = 5$, $\theta \in [0, 25]$

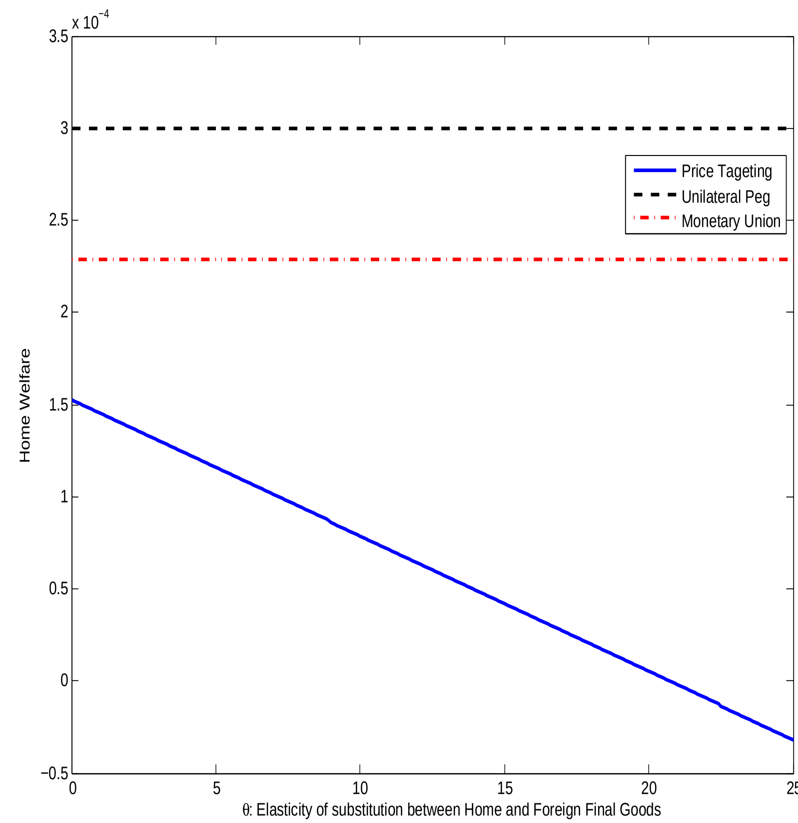


Figure 7: Home Welfare Comparison: $\theta = 0.5$, $\epsilon \in [0, 25]$

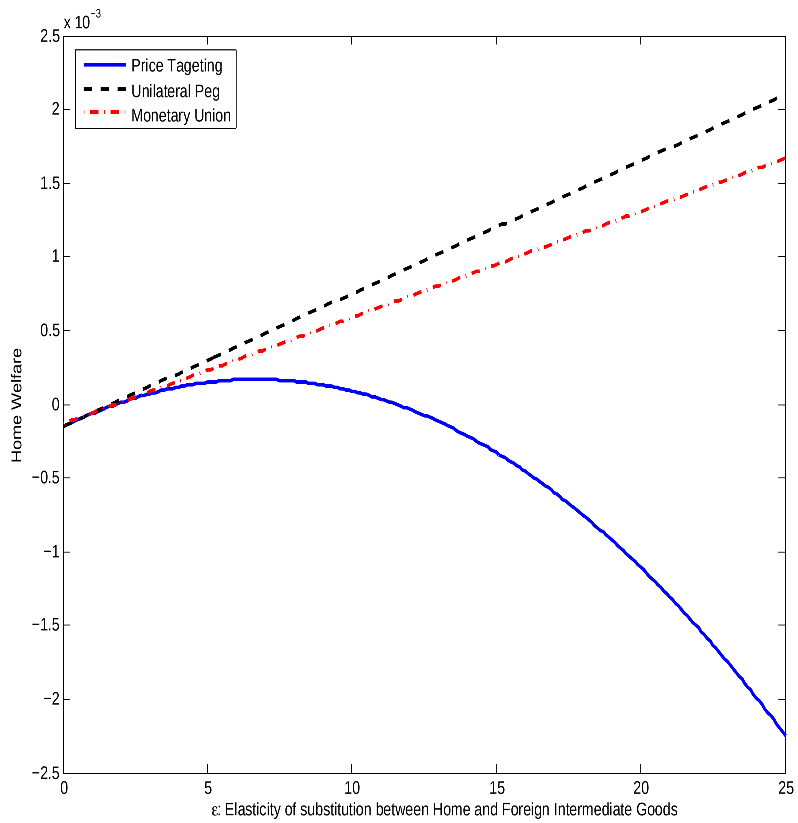


Figure 8: Home Welfare Comparison: $\theta = 1$, $\epsilon \in [0, 25]$

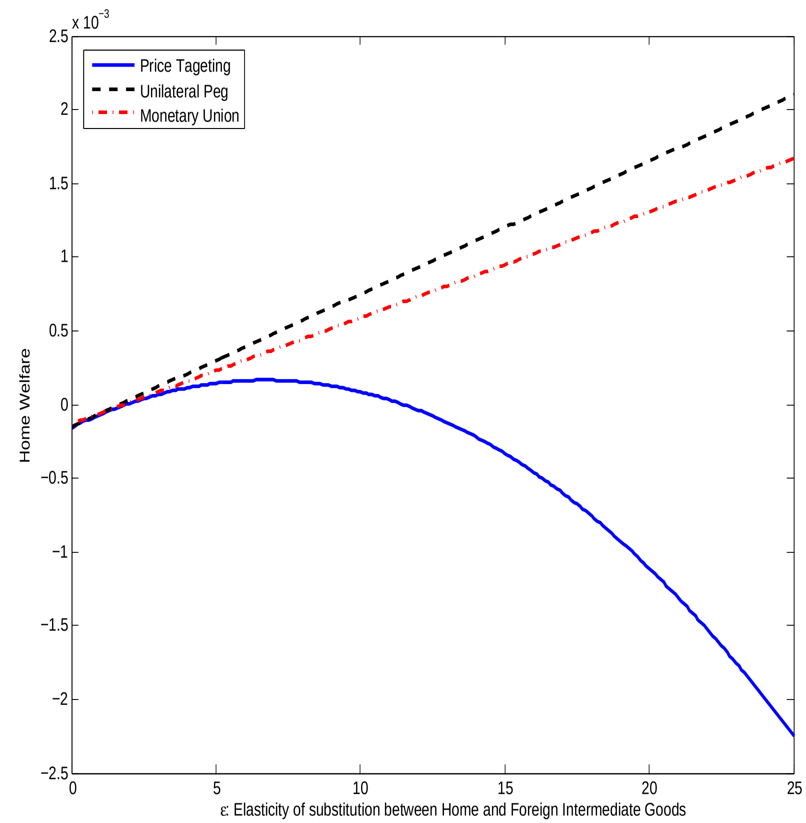


Figure 9: Home Welfare Comparison: $\theta = 2$, $\epsilon \in [0, 25]$

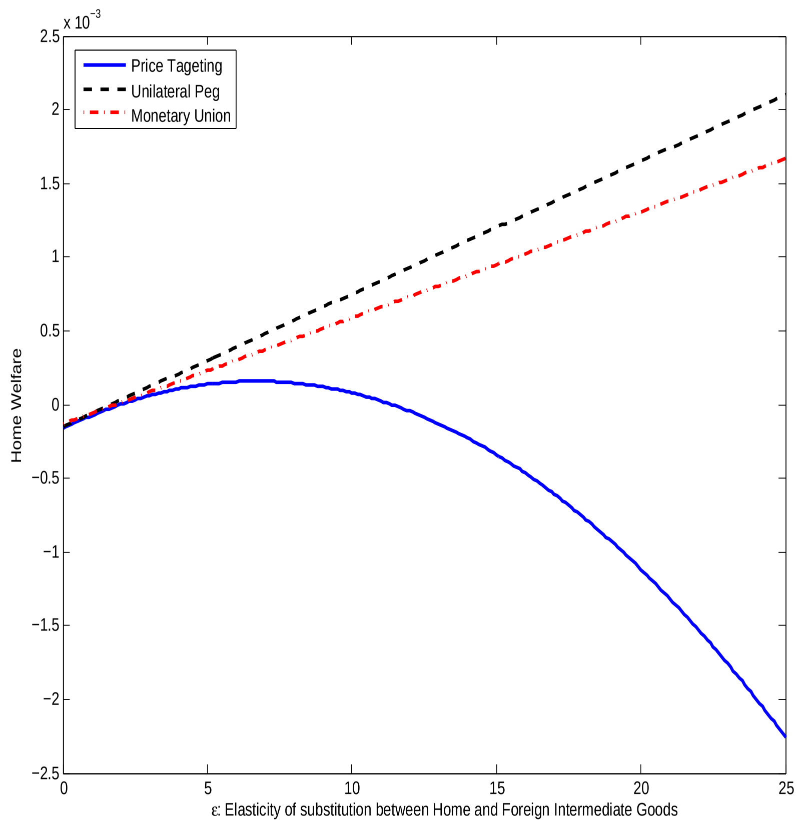


Figure 10: Home Welfare Comparison: $\theta = 5$, $\epsilon \in [0, 25]$

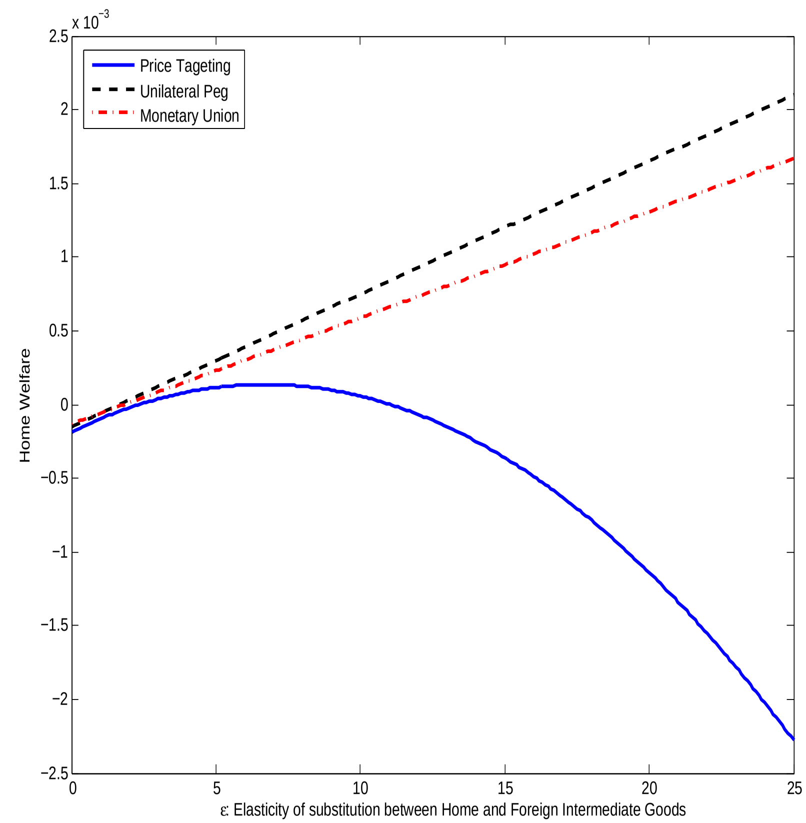


Figure 11: Foreign Welfare Comparison: $\epsilon = 0.5$, $\theta \in [0, 25]$

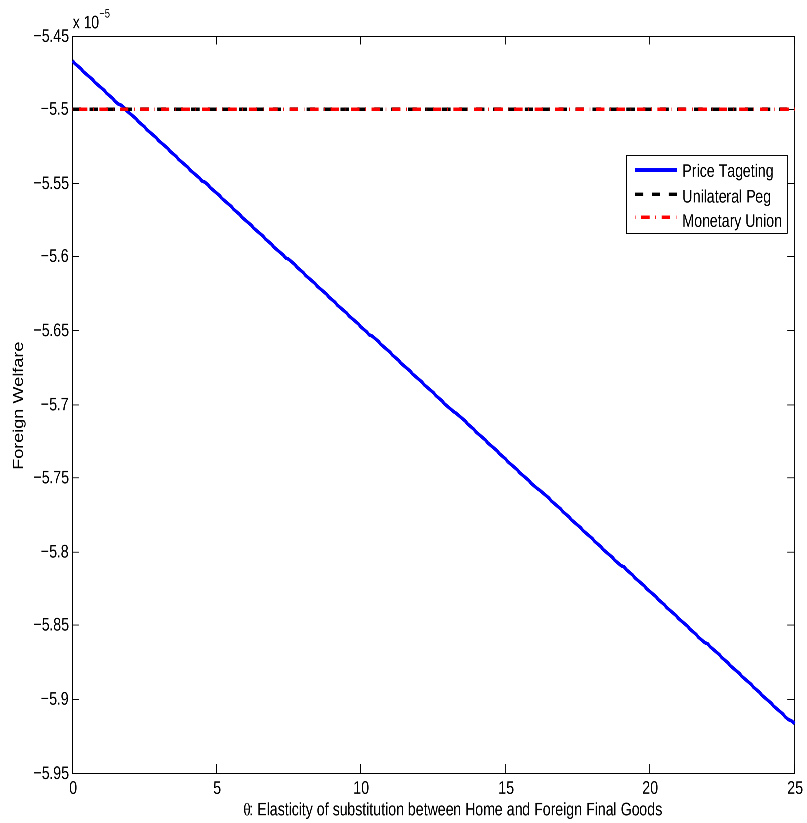


Figure 12: Foreign Welfare Comparison: $\epsilon = 1$, $\theta \in [0, 25]$

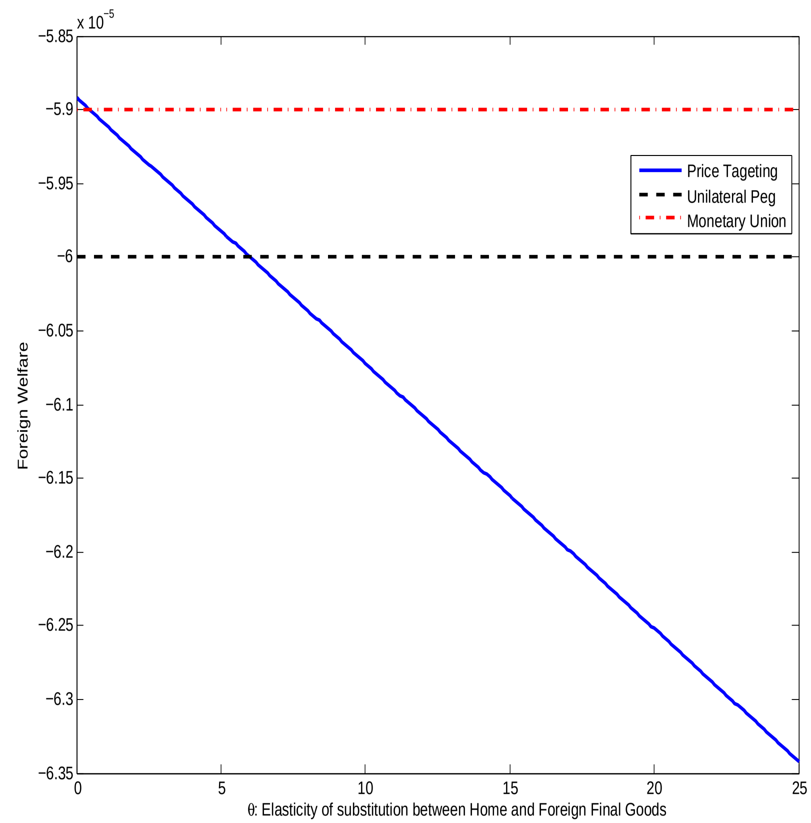


Figure 13: Foreign Welfare Comparison: $\epsilon = 2$, $\theta \in [0, 25]$

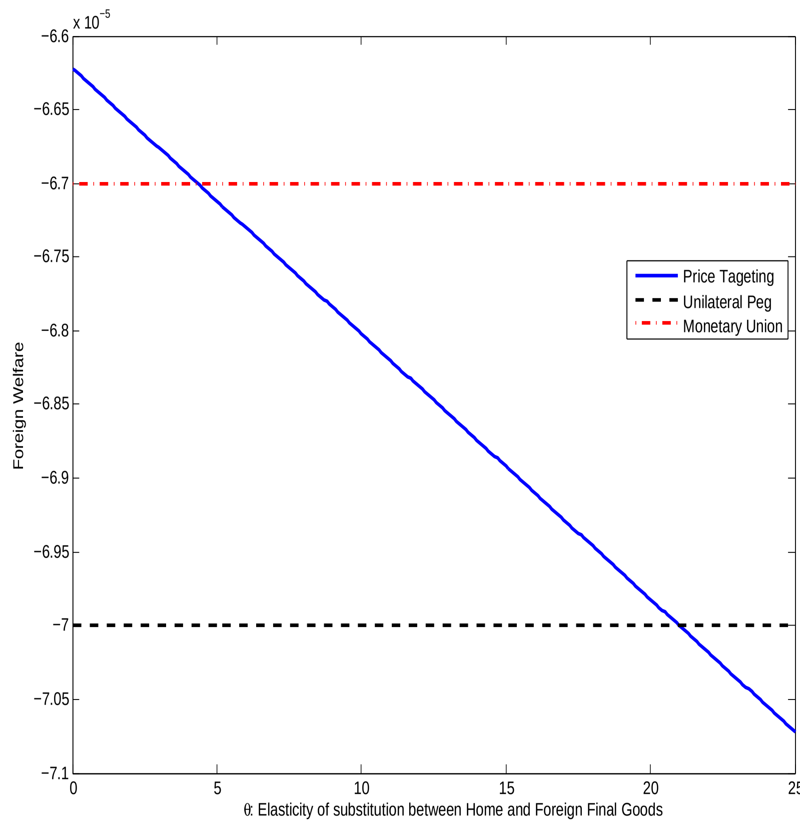


Figure 14: Foreign Welfare Comparison: $\epsilon = 5$, $\theta \in [0, 25]$

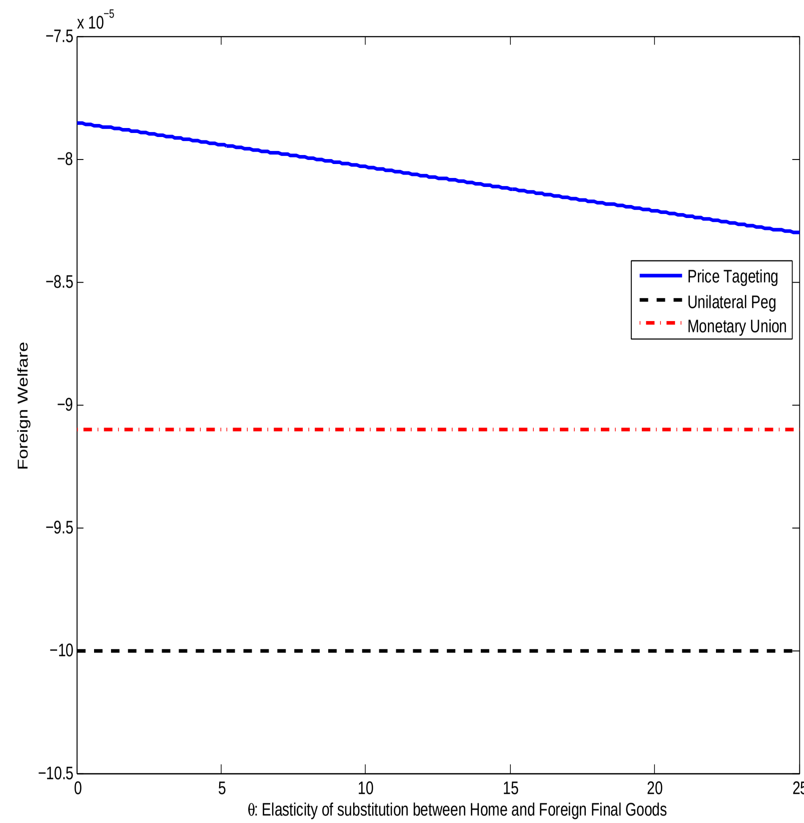


Figure 15: Foreign Welfare Comparison: $\theta = 0.5$, $\epsilon \in [0, 25]$

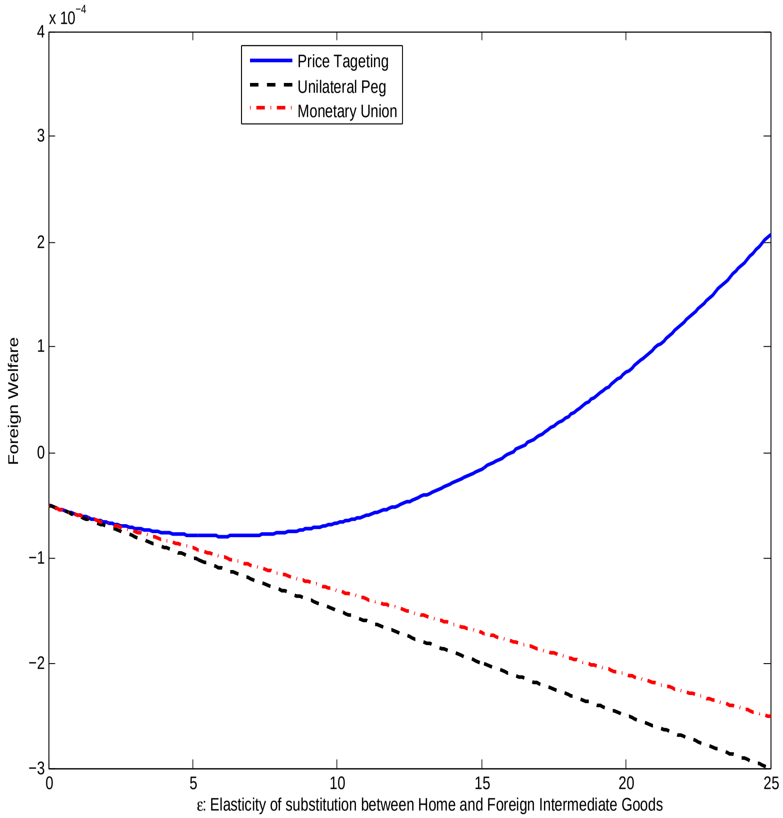


Figure 16: Foreign Welfare Comparison: $\theta = 1$, $\epsilon \in [0, 25]$

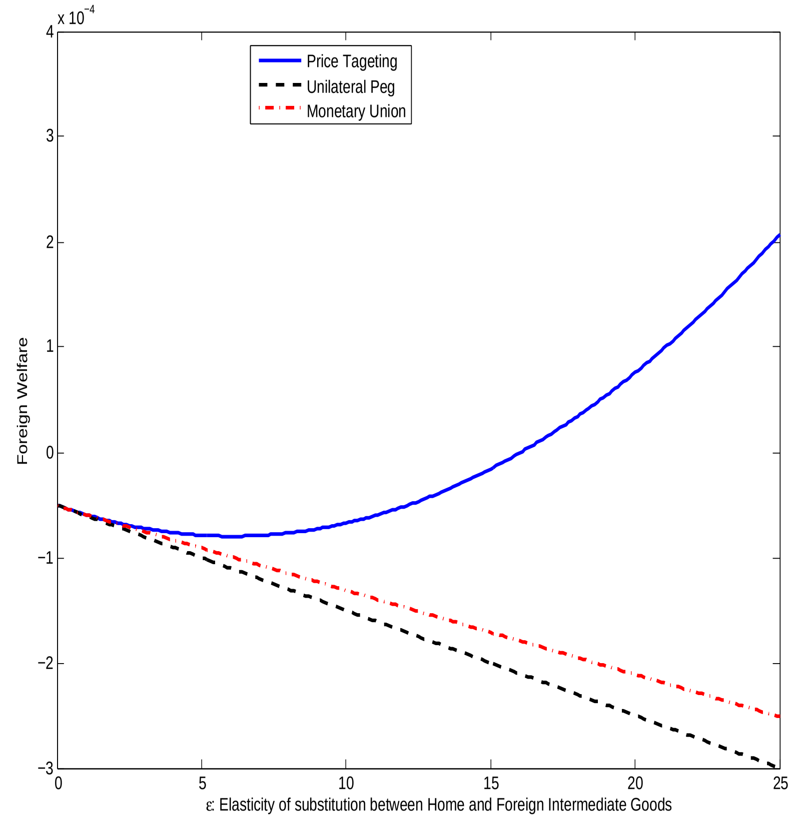


Figure 17: Foreign Welfare Comparison: $\theta = 2$, $\epsilon \in [0, 25]$

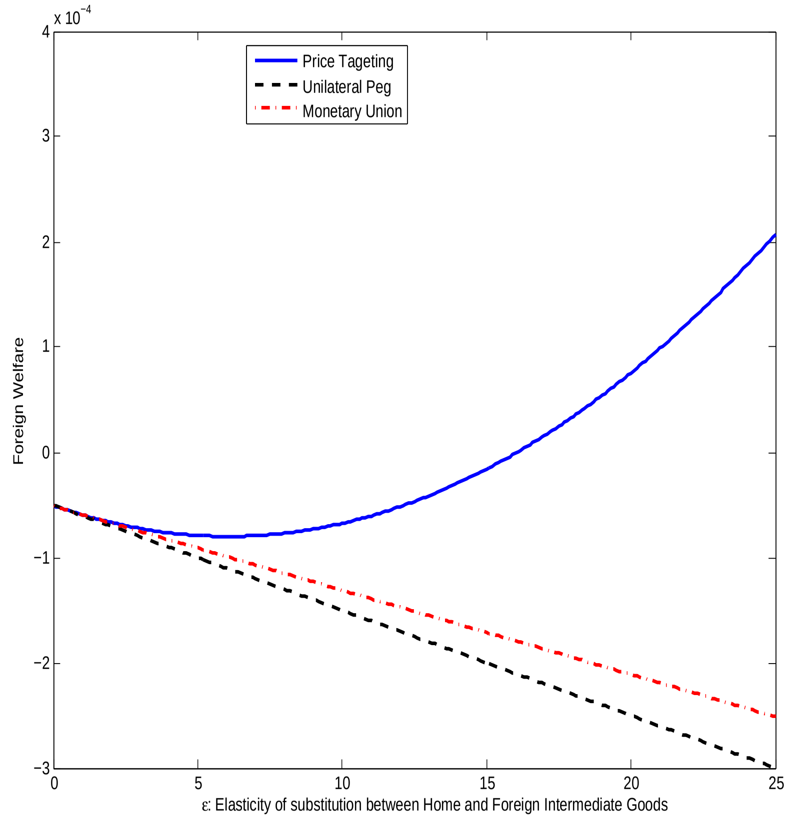
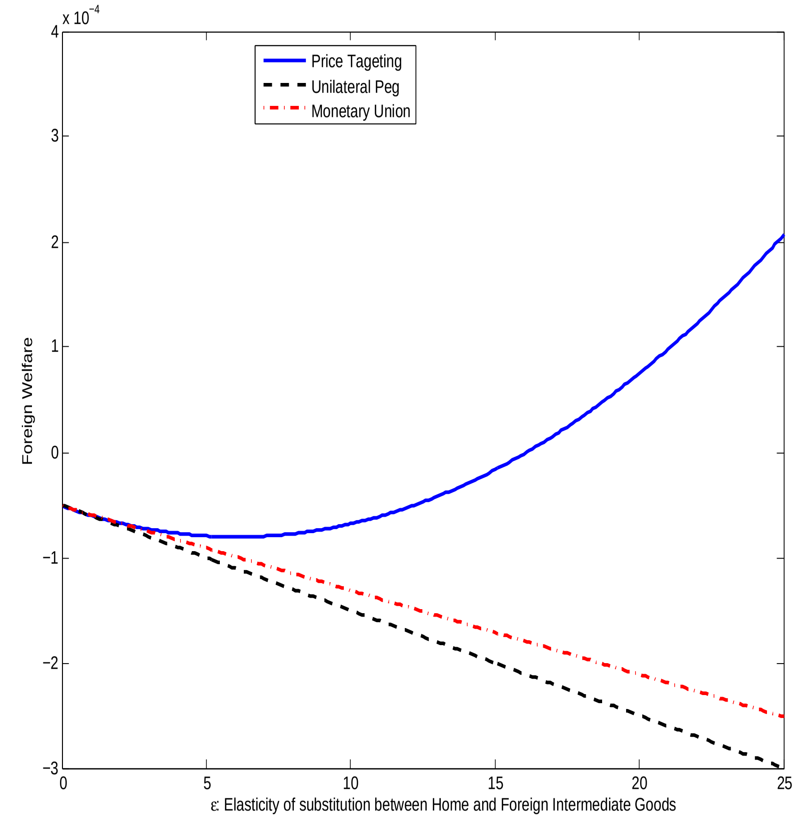


Figure 18: Foreign Welfare Comparison: $\theta = 5$, $\epsilon \in [0, 25]$



Technical Appendix of "Vertical Trade, Asymmetric Exchange Rate Pass-Through, and Canada's Exchange Rate Regime"

September 2011

(Not to be Published)

A.1 Log Linearization

The system consists of equations (5), (13)-(15), (19), (30), and their foreign counterparts, as well as (21)-(23), (27)-(29), and (37)-(38). Some of the equilibrium conditions are log-linear in themselves, such as the money demand equations, the labor supply equations, the risk-sharing condition, and the monetary policy rules.

$$\hat{M} = \hat{P} + \hat{C} \tag{A.1}$$

$$\hat{M}^* = \hat{P}^* + \hat{C}^* \tag{A.2}$$

$$\hat{W} = \hat{P} + \hat{C} + \psi \hat{L} \tag{A.3}$$

$$\hat{W}^* = \hat{P}^* + \hat{C}^* + \psi \hat{L}^* \tag{A.4}$$

$$\hat{P} + \hat{C} = \hat{S} + \hat{P}^* + \hat{C}^* \tag{A.5}$$

$$\hat{M} = -\delta_s \hat{S} - \delta_p \hat{P}^X \tag{A.6}$$

$$\hat{M}^* = -\delta_s^* \hat{S} - \delta_p^* \hat{P}^{*X} \tag{A.7}$$

First-order approximations of the other equilibrium conditions are given by

$$\hat{P} = n\hat{P}_H + (1-n)(\hat{P}_F^* + \hat{S}) + O(\epsilon^2) \tag{A.8}$$

$$\hat{P}^* = n\hat{P}_H^* + (1-n)\hat{P}_F^* + O(\epsilon^2) \tag{A.9}$$

$$\hat{\Lambda} = -\hat{A} + n\hat{\hat{P}}_H + (1-n)(\hat{\hat{P}}_F^* + \hat{S}) + O(\epsilon^2) \tag{A.10}$$

$$\hat{\Lambda}^* = -\hat{A}^* + n\hat{\tilde{P}}_H^* + (1-n)\hat{\tilde{P}}_F^* + O(\epsilon^2) \quad (A.11)$$

$$\hat{P}_H = \hat{P}_H^* = \hat{P}_F^* = \hat{\tilde{P}}_H = \hat{\tilde{P}}_H^* = \hat{\tilde{P}}_F^* = 0 + O(\epsilon^2) \quad (A.12)$$

$$\begin{aligned} \hat{L} &= -\hat{\tilde{A}} - n\left[\epsilon(\hat{\tilde{P}}_H - \hat{\Lambda}) + (1-\epsilon)\hat{A}\right] \\ &\quad -(1-n)\left[\epsilon(\hat{\tilde{P}}_H^* - \hat{\Lambda}^*) + (1-\epsilon)\hat{A}^*\right] + n\hat{C} + (1-n)\hat{C}^* + O(\epsilon^2) \quad (A.13) \end{aligned}$$

$$\begin{aligned} \hat{L}^* &= -\hat{\tilde{A}}^* - n\left[\epsilon(\hat{\tilde{P}}_F^* + \hat{S} - \hat{\Lambda}) + (1-\epsilon)\hat{A}\right] \\ &\quad -(1-n)\left[\epsilon(\hat{\tilde{P}}_F^* - \hat{\Lambda}^*) + (1-\epsilon)\hat{A}^*\right] + n\hat{C} + (1-n)\hat{C}^* + O(\epsilon^2) \quad (A.14) \end{aligned}$$

A.2 Figures for Section 4.3

Figure A.1 - Figure A.16 repeat the exercises in the paper by shutting down the final goods sector. In other words, Figure A.1 - Figure A.16 correspond to Figure (4) - (19) in the text but with $\sigma_A^2 = \sigma_{A^*}^2 = 0$ and $\sigma_{\tilde{A}}^2 = \sigma_{\tilde{A}^*}^2 = 0.0001$.

A.3 Final Goods PPI Inflation Targeting

Consider the following inflation targeting regime, in which both home and foreign central banks target the PPI of final goods, i.e., $P^X = \frac{\phi}{(\phi-1)(1+\gamma)}\Lambda$, $P^{*X} = \frac{\phi}{(\phi-1)(1+\gamma)}\Lambda^*$, $\delta_s = \delta_s^* = 0$, and $\delta_p = \delta_p^* \to \infty$. In this case, exchange rate will respond only to the final goods sector's productivity shocks and will create inefficient spillovers from the final goods sector to the intermediate goods sector. It first appears that the final goods PPI targeting works just like the intermediate goods PPI targeting where the welfare costs originate from the vertical chain of production and trade. However, these costs are much larger, indeed unbounded, when it is the intermediate goods sector that faces the negative spillovers. To explain this, let us look at the linearized model.

From equations (A.10)-(A.12), we have

$$\hat{\Lambda} = -\hat{A} + (1-n)\hat{S} + O(\epsilon^2) \quad (A.15)$$

$$\hat{\Lambda}^* = -\hat{A}^* + O(\epsilon^2) \quad \text{(A.16)}$$

To replicate the flexible prices of final goods, strict final goods PPI inflation targeting requires

$$\hat{M} = -\delta_p[-\hat{A} + (1-n)\hat{S}] + O(\epsilon^2) \quad \text{(A.17)}$$

$$\hat{M}^* = -\delta_p^*(-\hat{A}^*) + O(\epsilon^2) \quad \text{(A.18)}$$

Moreover, from equations (A.1)-(A.4), we know

$$\hat{W} = \hat{M} \quad \text{(A.19)}$$

$$\hat{W}^* = \hat{M}^* \quad \text{(A.20)}$$

That is to say, wages in each country will respond infinitely to the final goods sector's productivity shocks, which is inefficient since labor is only used in the intermediate goods sector not in the final goods sector. Such enormous fluctuations in wages, as well as in the unit costs of producing intermediate goods, create huge uncertainties in the economy. Labor supply becomes much more volatile, and intermediate goods producers ask for much higher risk premiums when they pre-set the prices. As a result, households are expected to consume less, work more, and endure massive volatilities in labor supply. Welfare for both countries are a lot lower than those under the final goods PPI inflation targeting. In fact, the stricter the final goods PPI inflation targeting (i.e., the higher δ_p and δ_p^* are), the lower the welfare for each country. As $\delta_p = \delta_p^* \to \infty$, the welfare for both countries approach negative infinity. This is why we choose not to focus on this type of policy regime in the paper.

Figure A.1: Home Welfare Comparison: $\epsilon = 0.5$, $\theta \in [0, 25]$

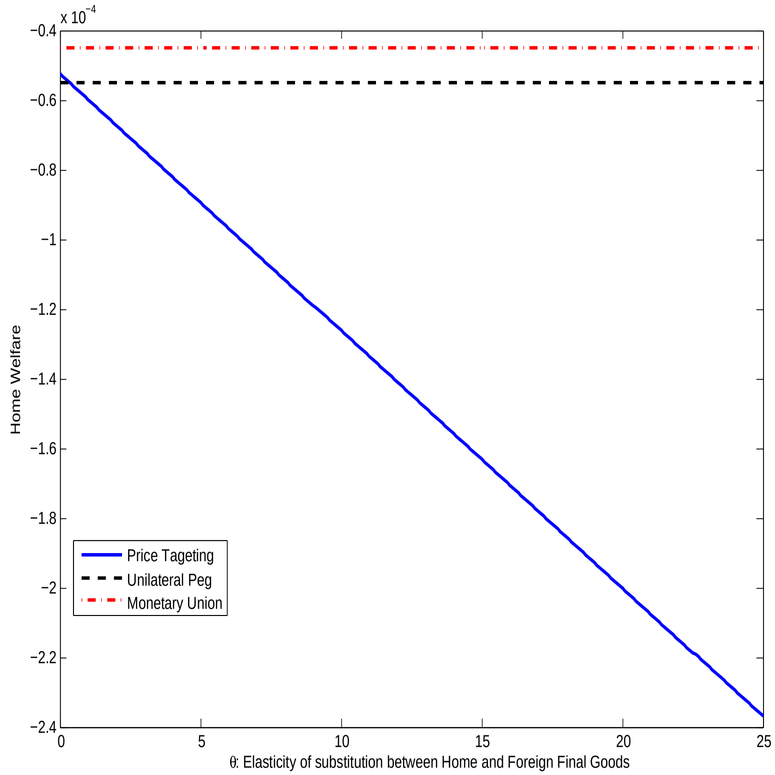


Figure A.2: Home Welfare Comparison: $\epsilon = 1$, $\theta \in [0, 25]$

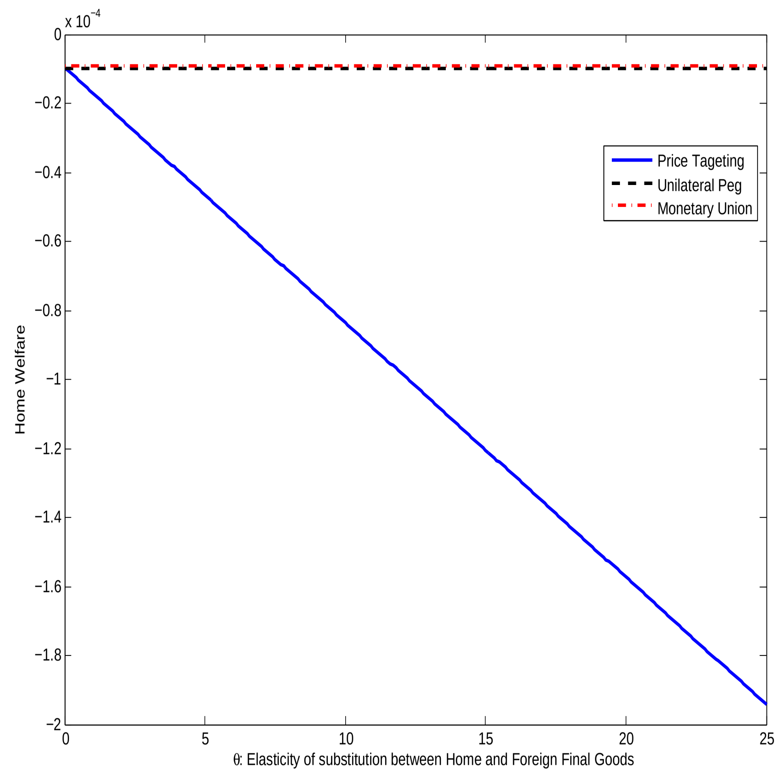


Figure A.3: Home Welfare Comparison: $\epsilon = 2$, $\theta \in [0, 25]$

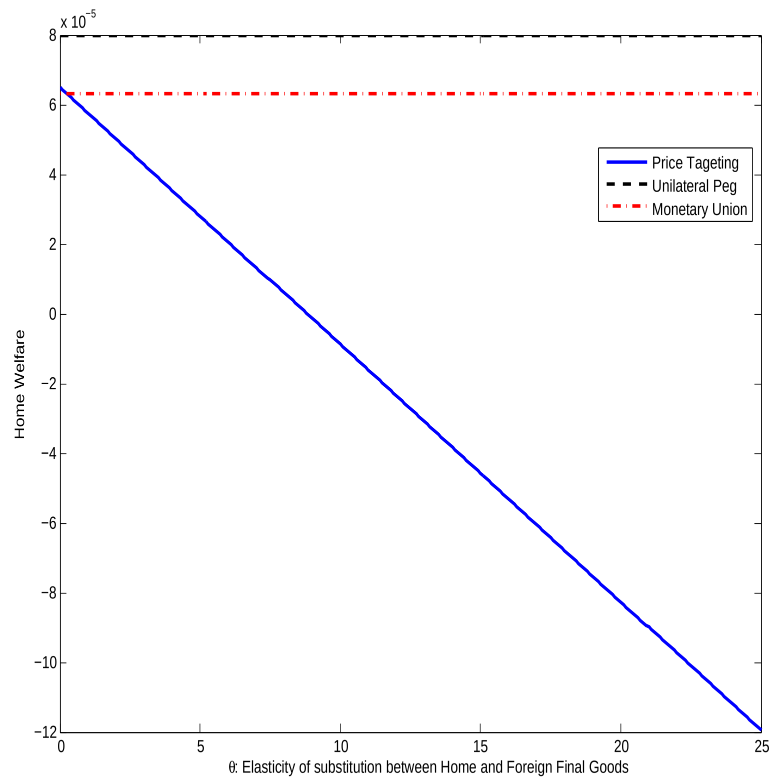


Figure A.4: Home Welfare Comparison: $\epsilon = 5$, $\theta \in [0, 25]$

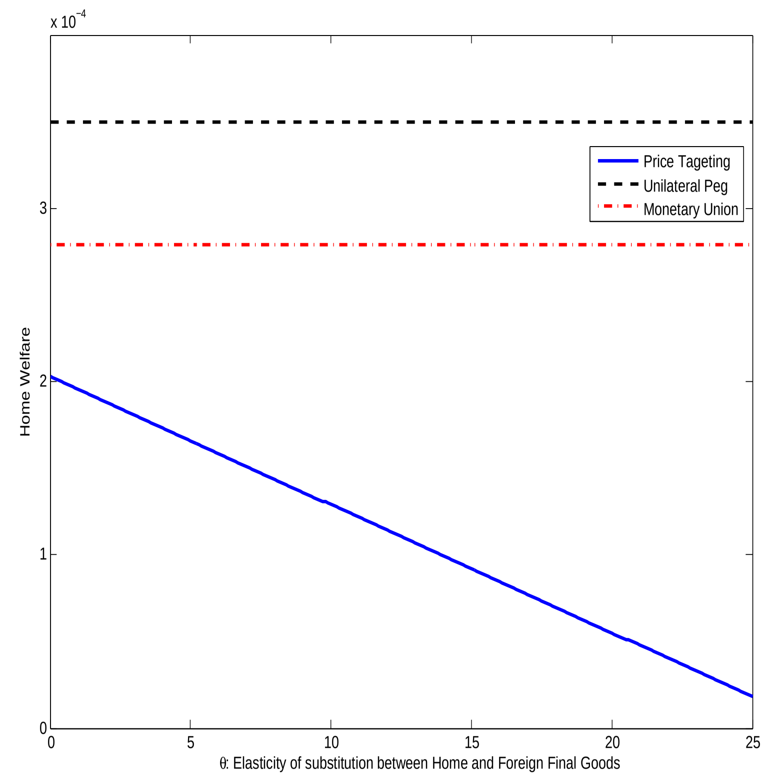


Figure A.5: Home Welfare Comparison: $\theta = 0.5$, $\epsilon \in [0, 25]$

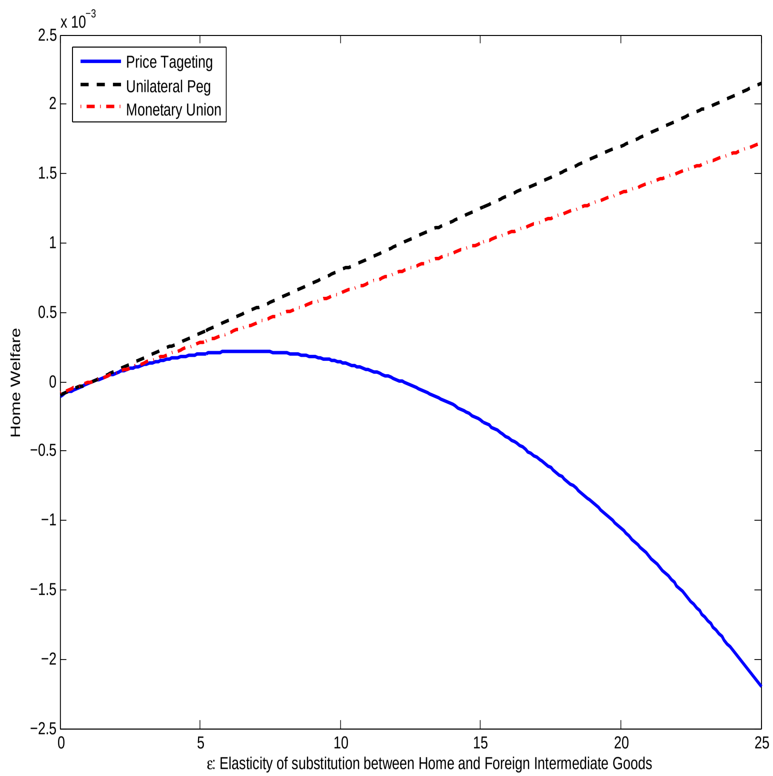


Figure A.6: Home Welfare Comparison: $\theta = 1$, $\epsilon \in [0, 25]$

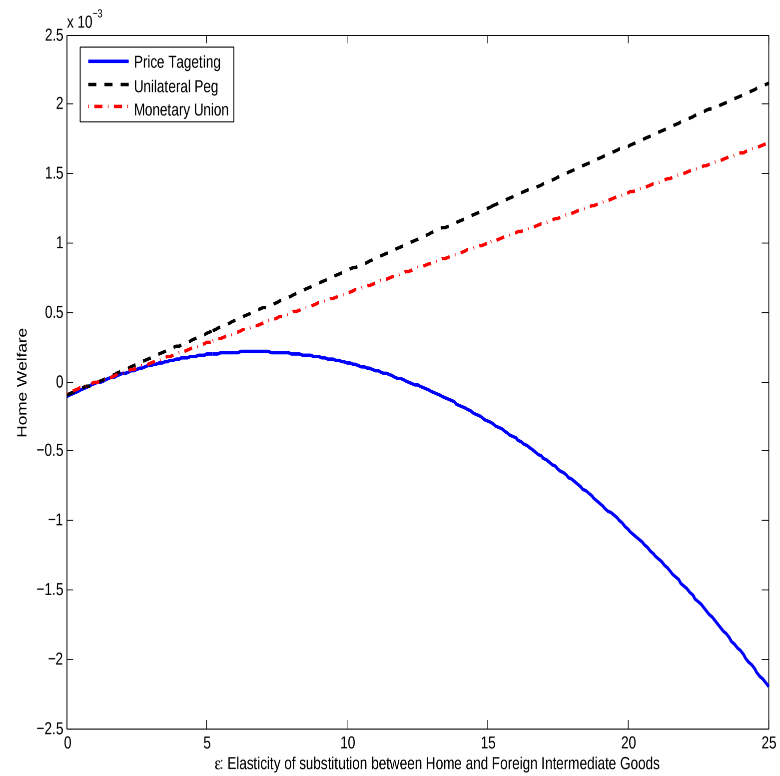


Figure A.7: Home Welfare Comparison: $\theta = 2$, $\epsilon \in [0, 25]$

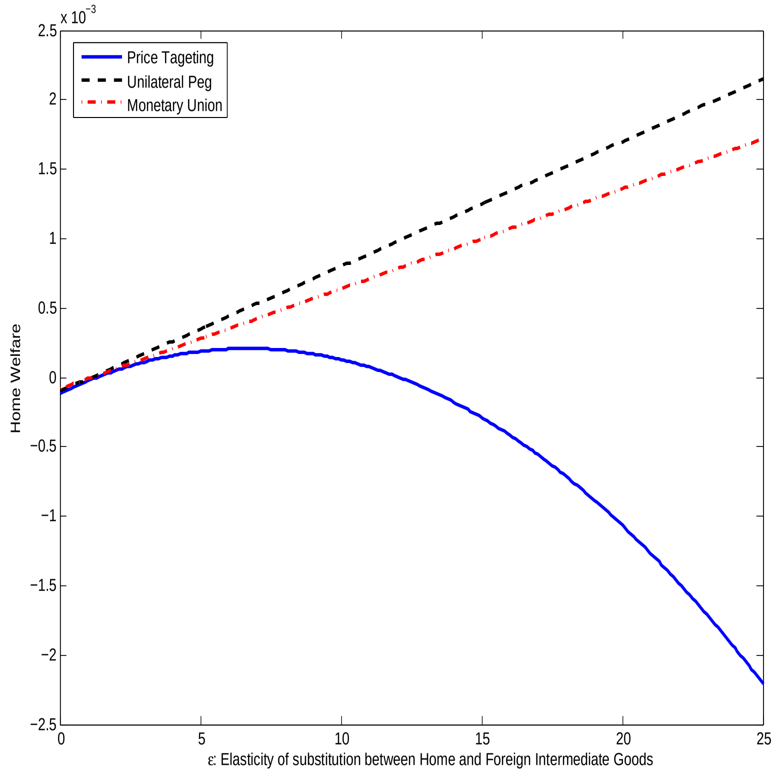


Figure A.8: Home Welfare Comparison: $\theta = 5$, $\epsilon \in [0, 25]$

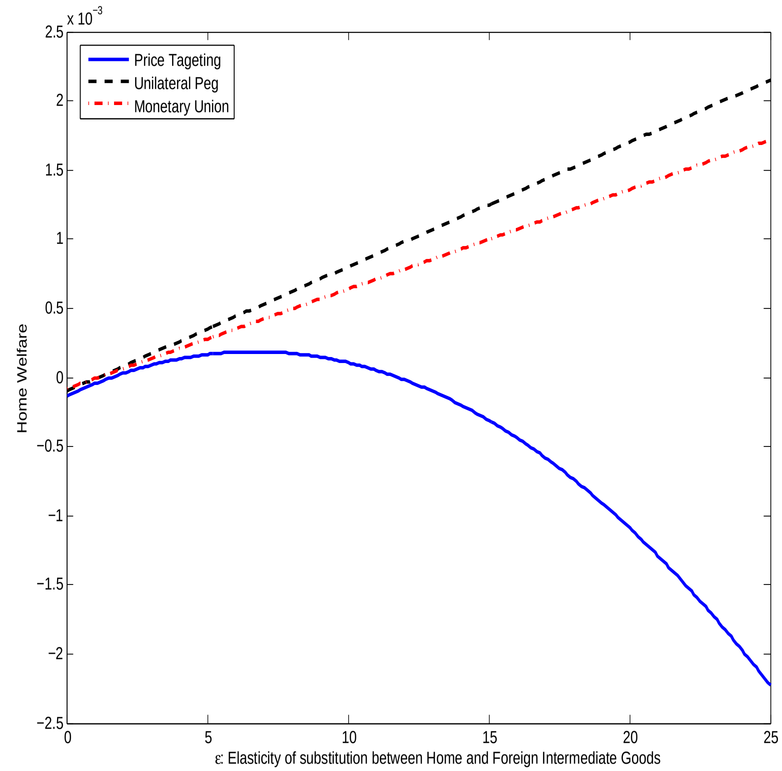


Figure A.9: Foreign Welfare Comparison: $\epsilon = 0.5$, $\theta \in [0, 25]$

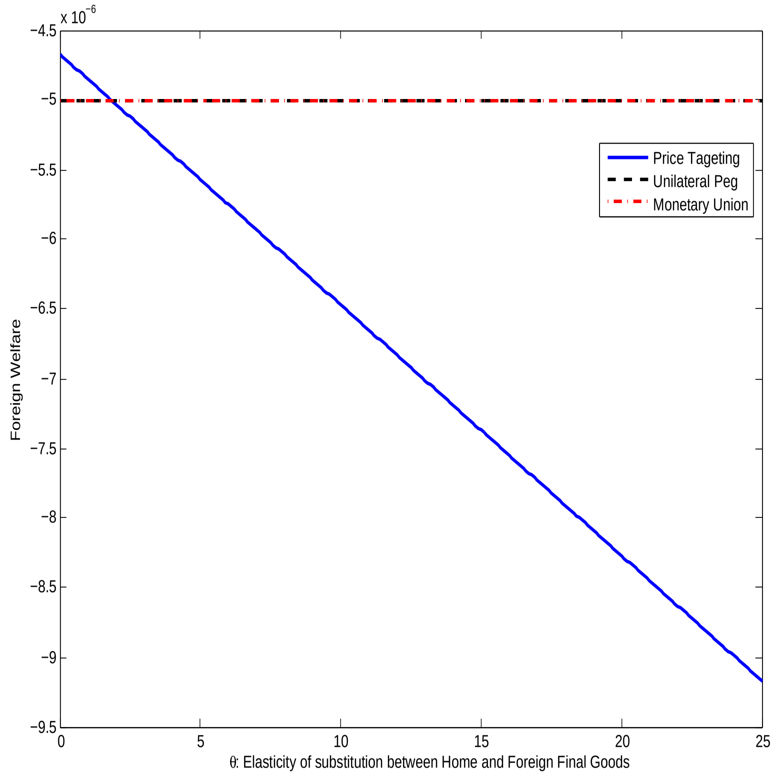


Figure A.10: Foreign Welfare Comparison: $\epsilon = 1$, $\theta \in [0, 25]$

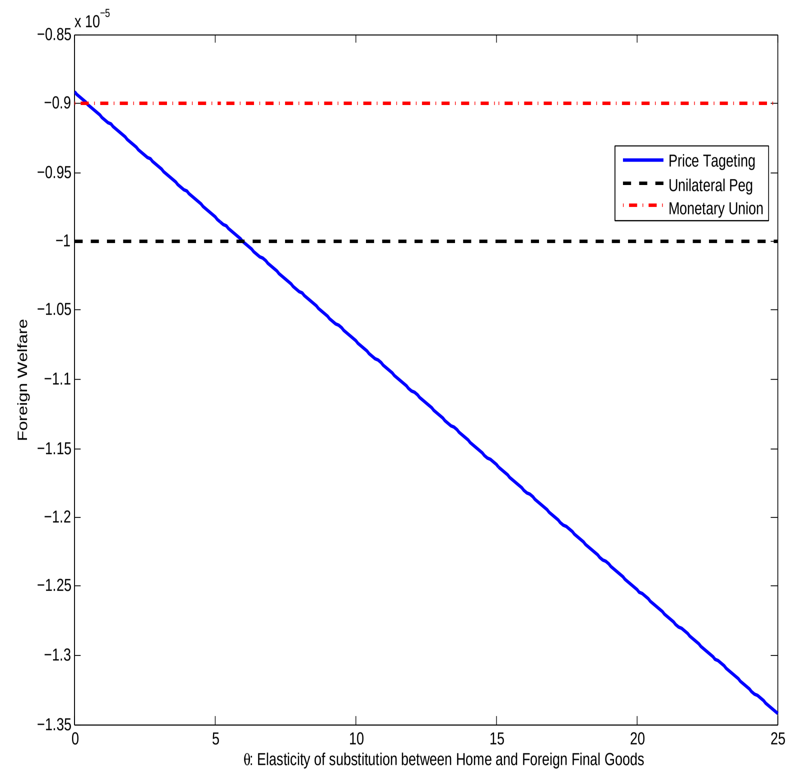


Figure A.11: Foreign Welfare Comparison: $\epsilon = 2$, $\theta \in [0, 25]$

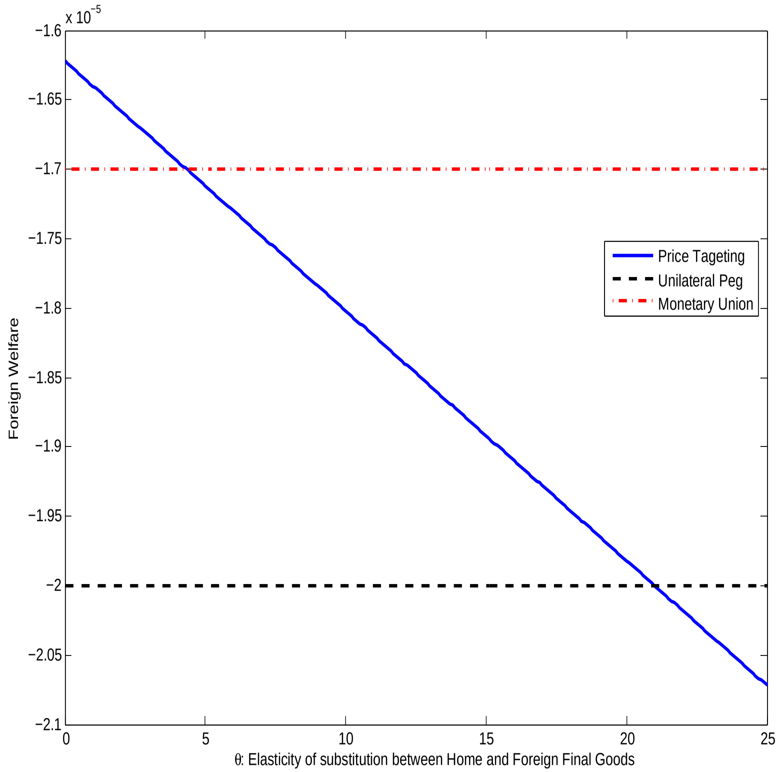


Figure A.12: Foreign Welfare Comparison: $\epsilon = 5$, $\theta \in [0, 25]$

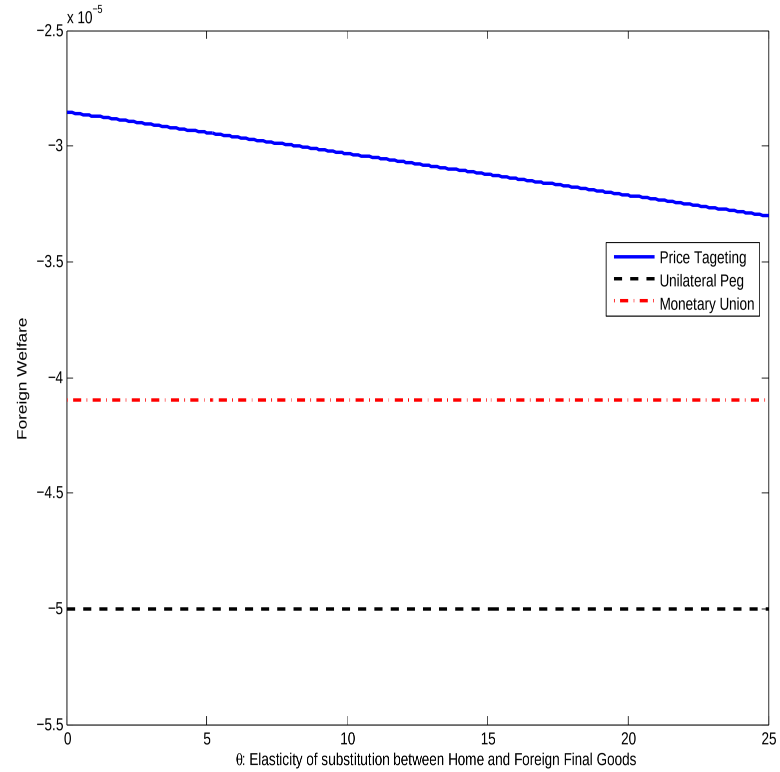


Figure A.13: Foreign Welfare Comparison: $\theta = 0.5$, $\epsilon \in [0, 25]$

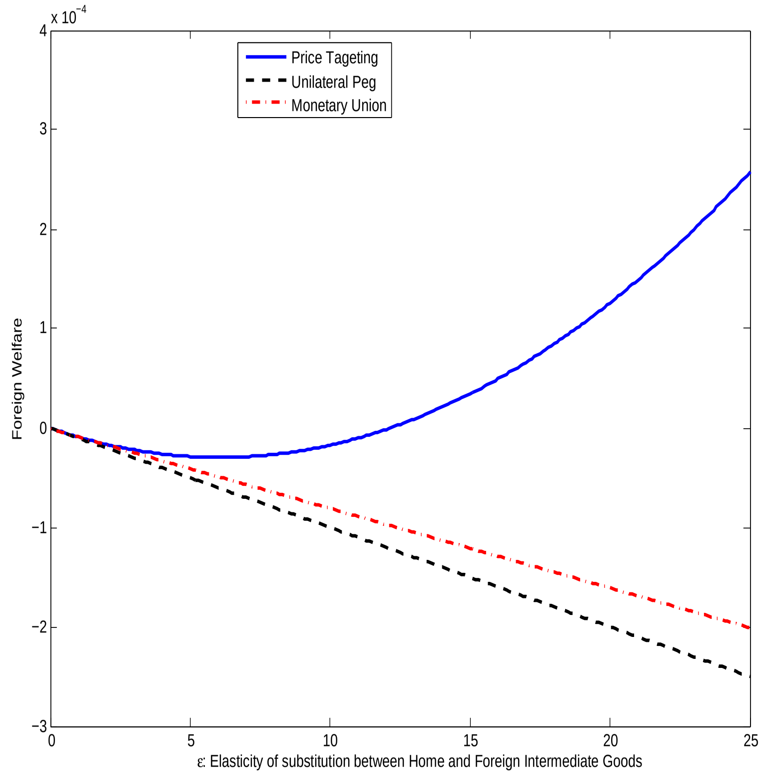


Figure A.14: Foreign Welfare Comparison: $\theta = 1$, $\epsilon \in [0, 25]$

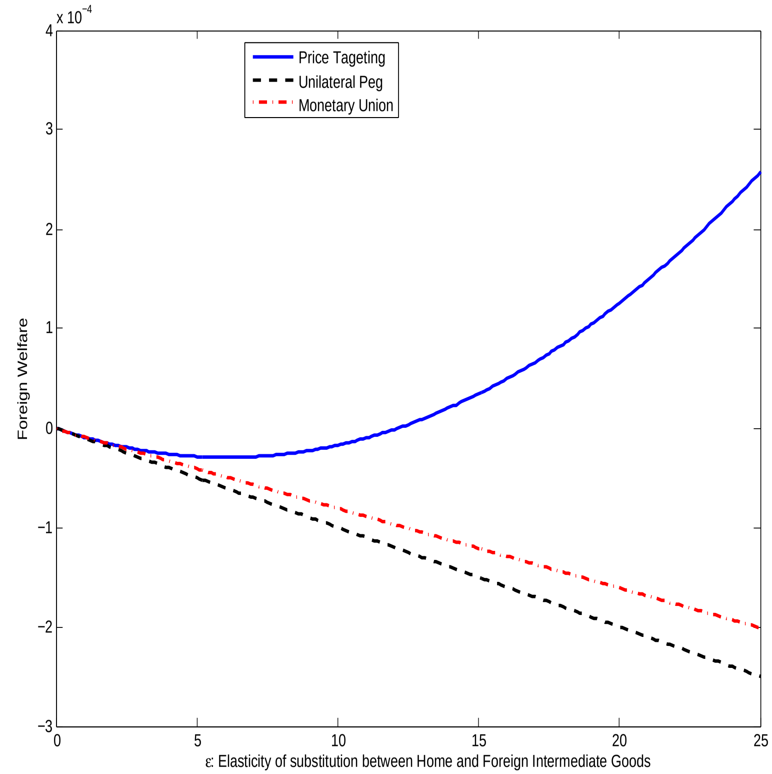


Figure A.15: Foreign Welfare Comparison: $\theta = 2$, $\epsilon \in [0, 25]$

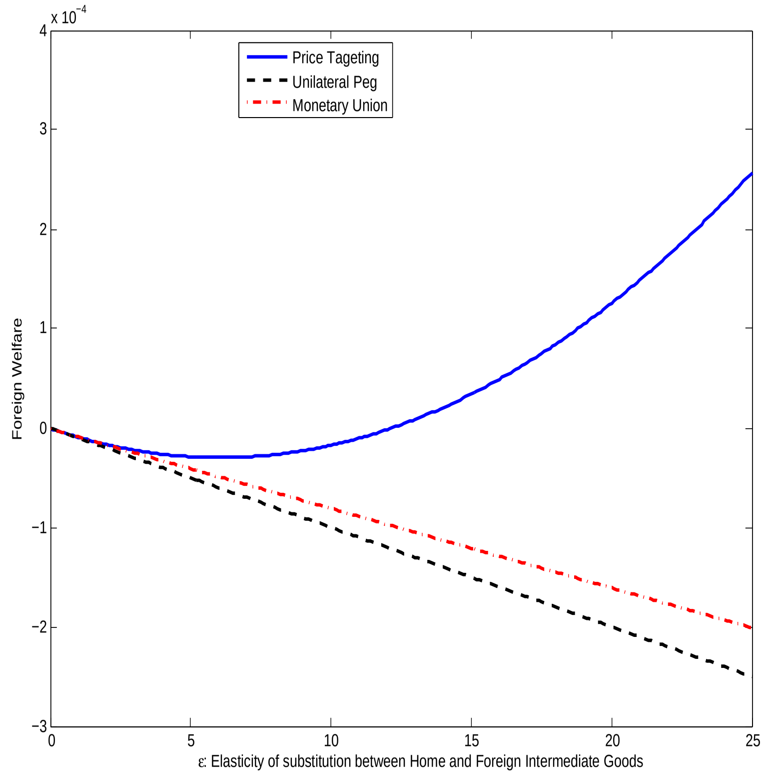


Figure A.16: Foreign Welfare Comparison: $\theta = 5$, $\epsilon \in [0, 25]$