Black Hole-Neutron Star Binaries in General Relativity

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Numerical Relativity

Solve Einstein’s equations

\[ G_{\mu\nu} = 8\pi T_{\mu\nu} \]

for spacetime metric (measures distances in 4D spacetime)

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij} (dx^i + \beta^i dt)(dx^j + \beta^j dt) \]

- Customary in numerical relativity: carve up spacetime into spatial slices (”3+1”)

Einstein’s equations split into

- **Constraint equations**
  - constrain \( \gamma_{ij} \) within each slice
  - solve to construct *initial data*

- **Evolution equations**
  - evolve \( \gamma_{ij} \) from one slice to next
  - solve to study *evolution*
Promising sources of gravitational radiation

Need huge time derivatives of huge quadrupole moments...

⇒ compact binaries
  [Baumgarte & Shapiro, 2002]

- Binary neutron stars
  - Initial data
    [Baumgarte et al., 1997; Uryū et al., 2000, Taniguchi & Gourgoulhon, 2003]
  - Evolution calculations
    [Shibata & Taniguchi, 2006]

- Binary black holes
  - Initial data
    [Cook & Pfeiffer, 2004]
  - Evolution calculations
    [Pretorius, 2005; Campanelli et al., 2005; Baker et al., 2005; Diener et al., 2005; Hermann et al., 2005]

- Black hole-neutron star (BHNS) binaries
  So far neglected!
BHNS binaries

- Promising sources of gravitational radiation
- Extremely rich astrophysically:
  Equate tidal force on test mass $m$ on surface of neutron star with gravitational force exerted by star

$$F_{\text{tid}} \sim G \frac{m M_{\text{BH}} R_{\text{NS}}}{s^3} \quad F_{\text{grav}} \sim G \frac{m M_{\text{NS}}}{R_{\text{NS}}^2}$$

- *tidal disruption* when

$$\frac{s_{\text{tid}}}{M_{\text{BH}}} \sim \left( \frac{M_{\text{NS}}}{M_{\text{BH}}} \right)^{2/3} \frac{R_{\text{NS}}}{M_{\text{NS}}}$$

- For neutron stars with $R_{\text{NS}}/M_{\text{NS}} \sim 5$ need $M_{\text{NS}}/M_{\text{BH}} \gtrsim 1/3$ for tidal disruption to occur outside innermost stable circular orbit at $r_{\text{ISCO}} \sim 6M_{\text{BH}}$
- But then $R_{\text{NS}} \gtrsim 1.6M_{\text{BH}}$
- Exact location and dynamics of break-up depend on equation of state
  $$\implies \text{Need fully relativistic dynamical simulations to understand tidal break-up!}$$

- Central engines of short GRBs? [See following talk by Josh Faber]
- Systematic study:
  - Initial data
  - Dynamical simulations
Initial Data

- Solve constraint equations to construct models of quasi-equilibrium binaries in quasi-circular orbit
- Constraint equations only specify subset of gravitational fields
  ⇒ hence choose decomposition and freely specifiable variables
  - Adopt conformal thin-sandwich decomposition for construction of quasi-equilibrium data [York, 1999]
  - Choose black-hole geometry as background geometry
    ⇒ takes care of black hole companion
- Take first integral of relativistic Euler equation to model equilibrium hydrodynamics
  \[ h \alpha \frac{\gamma}{\gamma_0} = \text{const} \]
Hierarchy of simplifying assumptions

• General binaries

• Assume extreme mass ratio $M_{\text{BH}} \gg M_{\text{NS}}$
  $\Rightarrow$ Center of rotation coincides with center of black hole
  $\Rightarrow$ Can restrict computational grid to neighborhood of neutron star

• Assume corotating matter flow
  $\Rightarrow$ Have $u^i = 0$ in corotating coordinate system
  $\Rightarrow$ Matter equations become \textit{algebraic}
Extreme Mass ratios / Corotating fluid

Can define relativistic Roche lobe from first integral of Euler equation

\[ \Rightarrow \text{Onset of tidal disruption when star fills Roche lobe} \]

Here \( M_{\text{BH}} = 10M_{\text{NS}} \) and \( n = 1.0 \)

[Baumgarte, Skoge & Shapiro, 2004]

\[ \hat{x}_{\text{BH}} = -5.0 \]

\[ \hat{x}_{\text{BH}} = -4.18 \]
Relax assumption of corotation

Allow for *irrotational* flow (more realistic astrophysically)

- can express spatial components of four-velocity in terms of velocity potential $\Phi$

\[ hu_i = D_i \Phi \]

\( \Rightarrow \) continuity equation becomes elliptic equation for $\Phi$

\( \Rightarrow \) tidal break-up occurs slightly *later* (i.e. at smaller separation) for irrotational binaries than for corotating binaries

[Taniguchi, Baumgarte, Faber & Shapiro, 2005]
Effect of background

Express black hole background in two different coordinate systems (Kerr-Schild and isotropic Schwarzschild)

⇒ different slicings lead to physical differences in conformal thin-sandwich formalism

How do we choose background in accordance with astrophysical expectation?

⇒ compare "waveless approximation"

[Uryū, Limousin, Friedman, Gourgoulhon & Shibata, 2005]
Relax assumption of extreme mass ratio

Now allow $M_{NS} \sim M_{BH}$

- Have to take into account effect of neutron star on black hole
  $\Rightarrow$ computational grid has to cover black hole
  $\Rightarrow$ impose equilibrium black hole boundary conditions
  [Cook & Pfeiffer, 2004]
- Need to locate center of rotation
  $\Rightarrow$ set linear momentum to zero

$\Rightarrow$ fully relativistic initial data for general BHNS binaries
[Taniguchi, Baumgarte, Faber & Shapiro, in preparation]
Dynamical simulations

• Similar hierarchy of simplifying assumptions

• So far:
  ○ extreme mass ratios
  ○ ”Wilson-Mathews”-approximation: keep spatial metric conformally flat
    [Faber, Baumgarte, Shapiro, Taniguchi & Rasio, 2006; Faber, Baumgarte, Shapiro & Taniguchi; in press]
  ○ great improvement over previous pseudo-Newtonian treatments
    \(\implies\) see Josh Faber’s talk

• current effort to relax these assumptions
Summary

• BHNS binaries very interesting
• Systematic study under way at Illinois/Bowdoin