Black Hole-Neutron Star Binaries in General Relativity

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Numerical Relativity

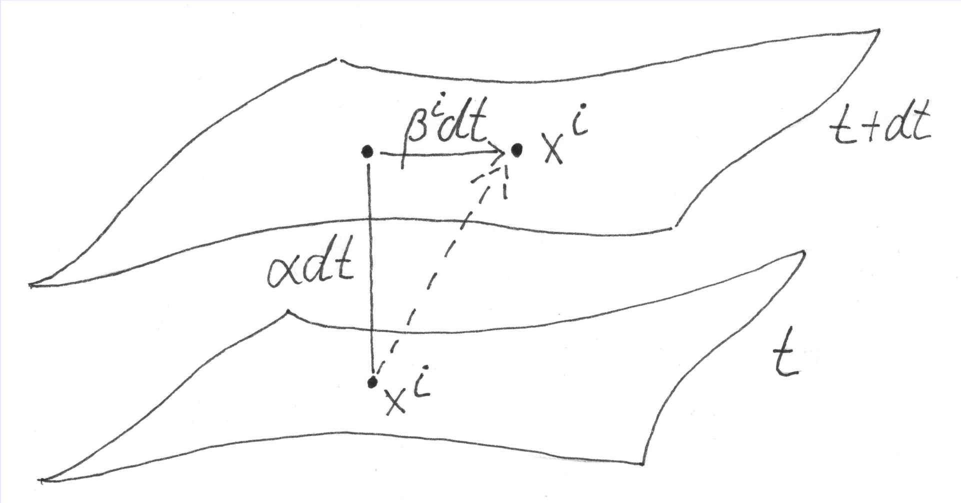
Solve Einstein's equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

for spacetime metric (measures distances in 4D spacetime)

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

- Customary in numerical relativity: carve up spacetime into spatial slices ("3+1")



Einstein's equations split into

Constraint equations
- constrain γ_{ij} within each slice
- solve to construct *initial data*

Evolution equations
- evolve γ_{ij} from one slice to next
- solve to study *evolution*

Promising sources of gravitational radiation

Need **huge** time derivatives of **huge** quadrupole moments...

$\Longrightarrow$ compact binaries
[Baumgarte & Shapiro, 2002]

- Binary neutron stars
  - Initial data
    [Baumgarte *et.al.*, 1997; Uryū *et.al.*, 2000, Taniguchi & Gourgoulhon, 2003]
  - Evolution calculations
    [Shibata & Taniguchi, 2006]
- Binary black holes
  - Initial data
    [Cook & Pfeiffer, 2004]
  - Evolution calculations
    [Pretorius, 2005; Campanelli *et.al.*, 2005; Baker *et.al.*, 2005; Diener *et.al.*, 2005; Hermann *et.al.*, 2005]
- Black hole-neutron star (BHNS) binaries

$\boxed{\text{So far neglected!}}$

BHNS binaries

- Promising sources of gravitational radiation
- Extremely rich astrophysically:
  Equate tidal force on test mass m on surface of neutron star with gravitational force exerted by star

$$F_{\rm tid} \sim G\frac{mM_{\rm BH}R_{\rm NS}}{s^3} \qquad\qquad F_{\rm grav} \sim G\frac{mM_{\rm NS}}{R_{\rm NS}^2}$$

  ◦ *tidal disruption* when

$$\frac{s_{\rm tid}}{M_{\rm BH}} \sim \left(\frac{M_{\rm NS}}{M_{\rm BH}}\right)^{2/3} \frac{R_{\rm NS}}{M_{\rm NS}}$$

  ◦ For neutron stars with $R_{\rm NS}/M_{\rm NS} \sim 5$ need $M_{\rm NS}/M_{\rm BH} \gtrsim 1/3$ for tidal disruption to occur outside innermost stable circular orbit at $r_{\rm ISCO} \sim 6M_{\rm BH}$
  ◦ But then $R_{\rm NS} \gtrsim 1.6M_{\rm BH}$
  ◦ Exact location and dynamics of break-up depend on equation of state
  $\Longrightarrow$ Need fully relativistic dynamical simulations to understand tidal break-up!
- Central engines of short GRBs? [See following talk by Josh Faber]
- Systematic study:
  ◦ Initial data
  ◦ Dynamical simulations

Initial Data

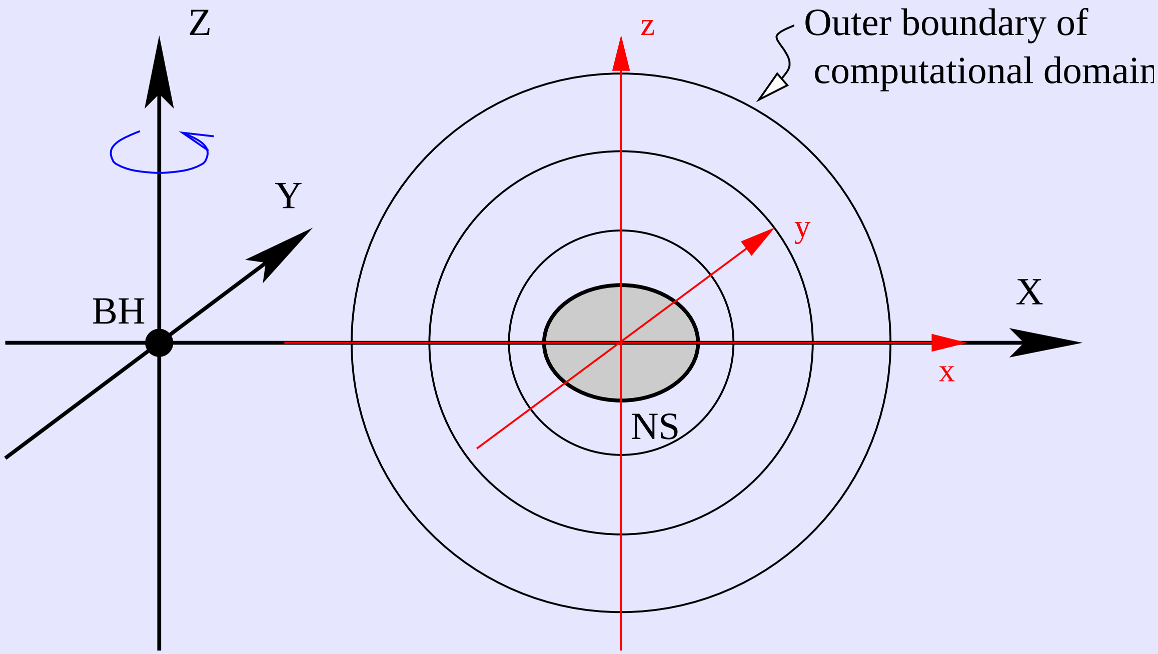
- Solve constraint equations to construct models of quasi-equilibrium binaries in quasi-circular orbit
- Constraint equations only specify subset of gravitational fields
  $\Longrightarrow$ hence *choose* decomposition and freely specifiable variables
  - Adopt conformal thin-sandwich decomposition for construction of quasi-equilibrium data [York, 1999]
  - Choose black-hole geometry as background geometry
    $\Longrightarrow$ takes care of black hole companion
- Take first integral of relativistic Euler equation to model equilibrium hydrodynamics

$$h\alpha\frac{\gamma}{\gamma_0} = \text{const}$$

Hierarchy of simplifying assumptions

- General binaries

- Assume extreme mass ratio $M_{\mathrm{BH}} \gg M_{\mathrm{NS}}$
  $\Longrightarrow$ Center of rotation coincides with center of black hole
  $\Longrightarrow$ Can restrict computational grid to neighborhood of neutron star

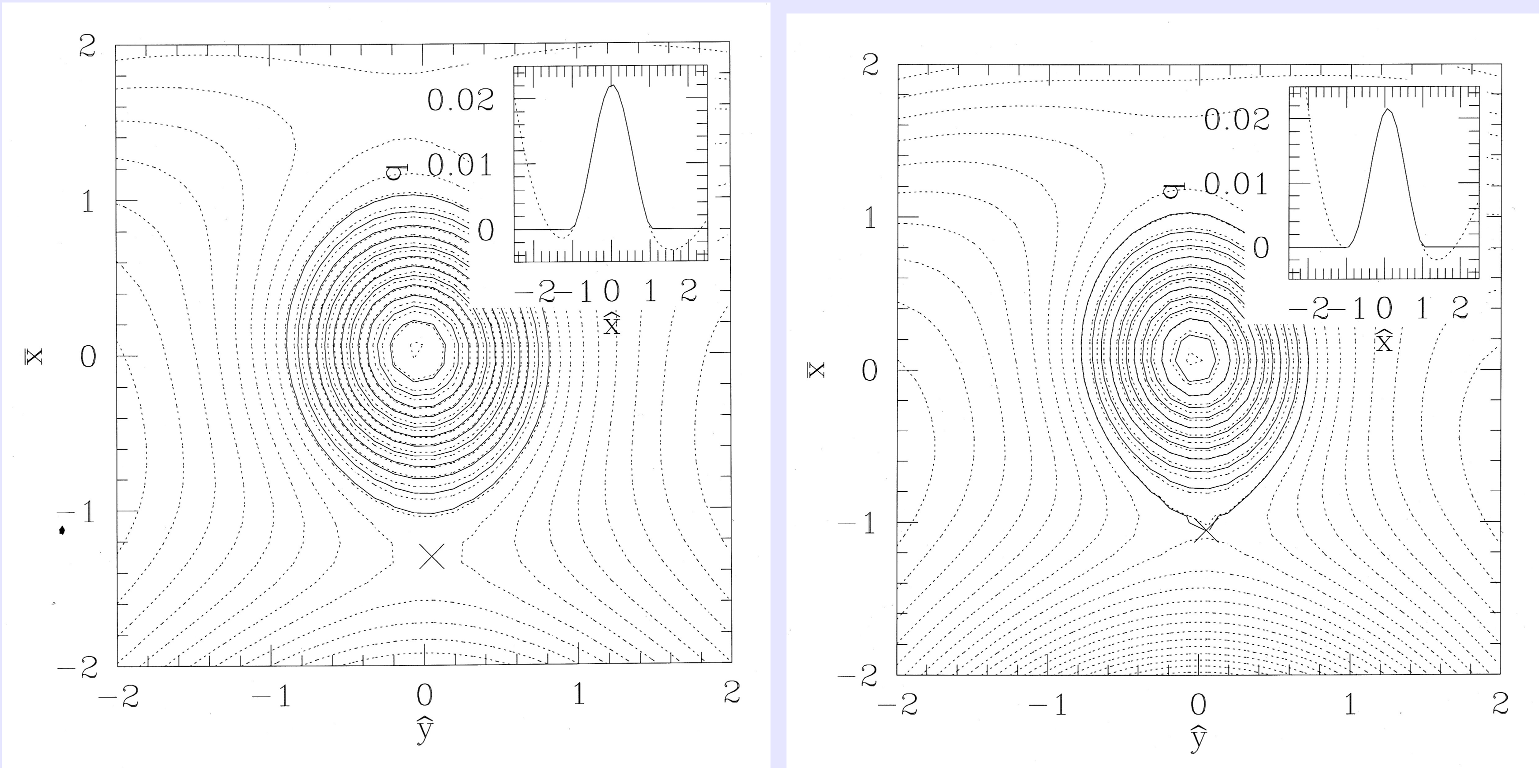


- Assume corotating matter flow
  $\Longrightarrow$ Have $u^i = 0$ in corotating coordinate system
  $\Longrightarrow$ Matter equations become *algebraic*

Extreme Mass ratios / Corotating fluid

Can define relativistic Roche lobe from first integral of Euler equation

$\Longrightarrow$ Onset of tidal disruption when star fills Roche lobe



$\hat{x}_{\rm BH} = -5.0$ $\qquad$ $\hat{x}_{\rm BH} = -4.18$

Here $M_{\rm BH} = 10 M_{\rm NS}$ and $n = 1.0$

[Baumgarte, Skoge & Shapiro, 2004]

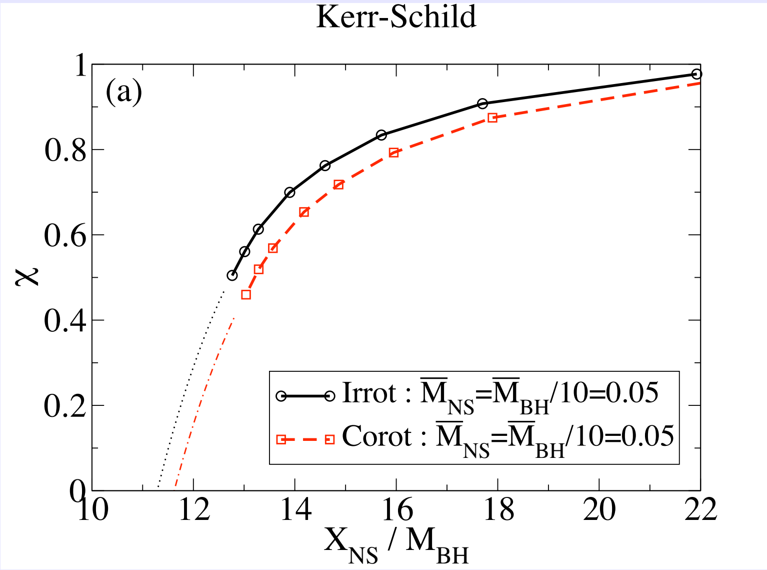
Relax assumption of corotation

Allow for *irrotational* flow (more realistic astrophysically)

- can express spatial components of four-velocity in terms of velocity potential Φ

$$hu_i = D_i \Phi$$

$\Longrightarrow$ continuity equation becomes elliptic equation for Φ



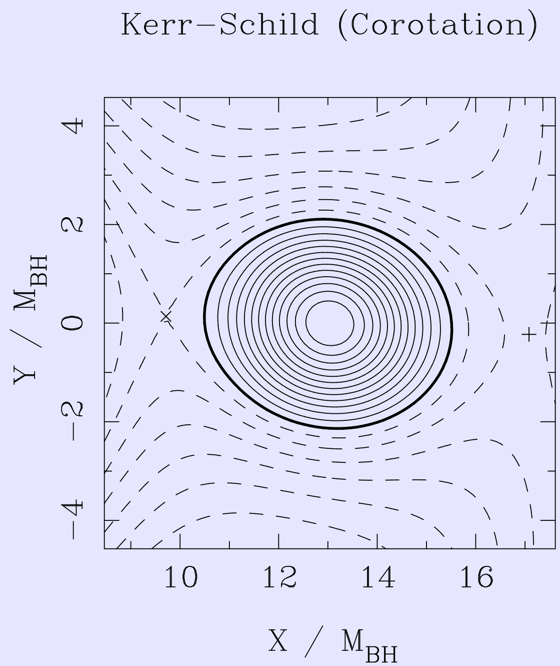
$\Longrightarrow$ tidal break-up occurs slightly *later* (i.e. at smaller separation) for irrotational binaries than for corotating binaries

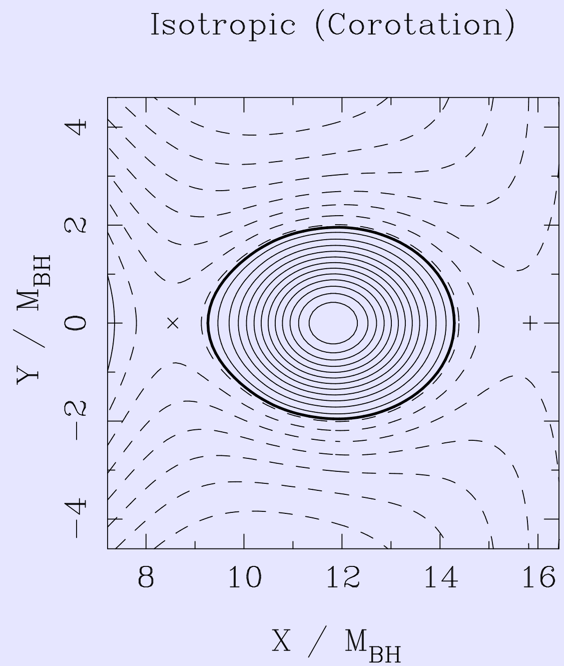
[Taniguchi, Baumgarte, Faber & Shapiro, 2005]

Effect of background

Express black hole background in two different coordinate systems (Kerr-Schild and isotropic Schwarzschild)

$\Longrightarrow$ different slicings lead to physical differences in conformal thin-sandwich formalism





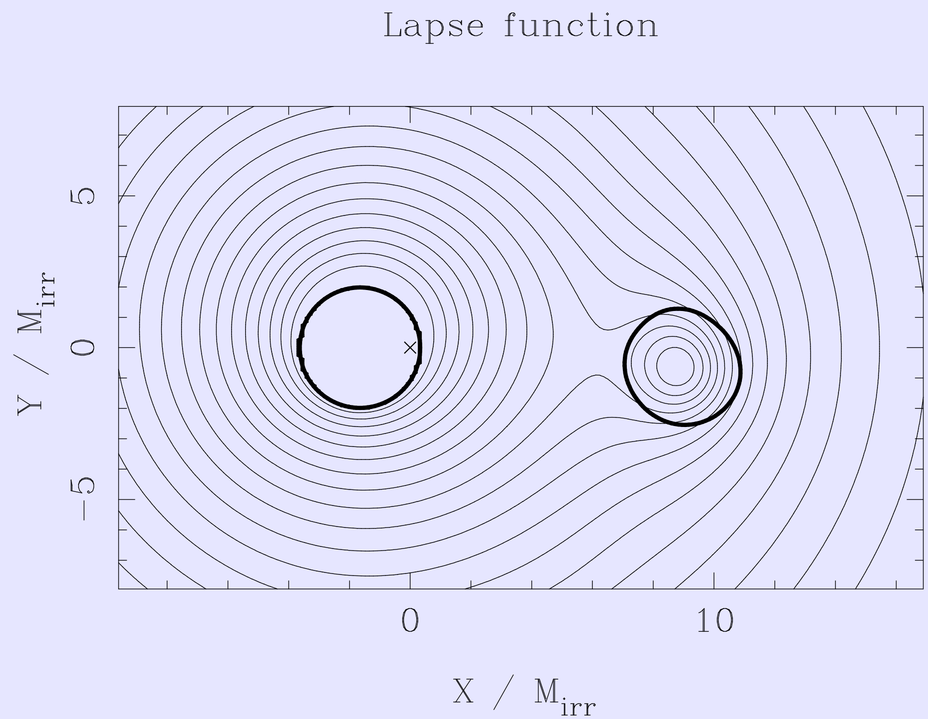
$\Longrightarrow$ How do we choose background in accordance with astrophysical expectation?

$\Longrightarrow$ compare "waveless approximation"
[Uryū, Limousin, Friedman, Gourgoulhon & Shibata, 2005]

Relax assumption of extreme mass ratio

Now allow $M_{\mathrm{NS}} \sim M_{\mathrm{BH}}$

- Have to take into account effect of neutron star on black hole
  $\Longrightarrow$ computational grid has to cover black hole
  $\Longrightarrow$ impose equilibrium black hole boundary conditions
  [Cook & Pfeiffer, 2004]
- Need to locate center of rotation
  $\Longrightarrow$ set linear momentum to zero



$\Longrightarrow$ fully relativistic initial data for general BHNS binaries
[Taniguchi, Baumgarte, Faber & Shapiro, in preparation]

Dynamical simulations

- Similar hierarchy of simplifying assumptions
- So far:
  - extreme mass ratios
  - "Wilson-Mathews"-approximation: keep spatial metric conformally flat

  [Faber, Baumgarte, Shapiro, Taniguchi & Rasio, 2006; Faber, Baumgarte, Shapiro & Taniguchi; in press]
  - great improvement over previous pseudo-Newtonian treatments

  $\Longrightarrow$ see Josh Faber's talk
- current effort to relax these assumptions

Summary

- BHNS binaries very interesting
- Systematic study under way at Illinois/Bowdoin