CSci 231 Homework 2

Growth of Functions, Summations and Recurrences

CLRS Chapter 3, 4 and Appendix A

Write and justify your answers on this sheet in the space provided.[1]

1. (CLRS 3-2)

[1]Collaboration is allowed, even encouraged, provided that the names of the collaborators are listed along with the solutions. Write up the solutions on your own.

2. (CLRS A.1-1) Find a simple formula for $\sum_{k=1}^{n}(2k-1)$.

3. Prove by induction that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

4. Solve the recurrence: $T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(n-1) + n(n-1) & \text{if } n \geq 2 \end{cases}$

   Hint: use $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

5. CLRS 3-4 part (a) only.

6. (CLRS 4.2-2) Argue that the solution to the recurrence

$$T(n) = T(n/3) + T(2n/3) + n$$

is $\Omega(n \log n)$ by appealing to a recursion tree.

Give asymptotic upper and lower bounds for the following recurrences. Assume T(n) is constant for $n \leq 2$. Make your bounds as tight as possible, and justify your answers.

7. $T(n) = T(n-1) + n$

8. $T(n) = T(\sqrt{n}) + 1$

9. $T(n) = 2T(n/2) + n/\lg n$

10. $T(n) = T(n-1) + 1/n$

11. $T(n) = 2T(n/4) + \sqrt{n}$

12. $T(n) = 7T(n/2) + n^3$

13. $T(n) = 7T(n/2) + n^2$

14. $T(n) = 5T(n/5) + n/\log n$