

CPS 231 Exam 2

Fall 2003

1:00 - 2:25, Tuesday November 20th
Closed book exam

NAME: _____

Problem	Max	Obtained
1	10	
2	25	
3 (a)	15	
3 (b)	15	
3 (c)	10	
4 (a)	10	
4 (b)	15	
4 (c)	10	
Total	110	

[10 points] **Problem 1:**

1. Is it true that in the worst case, a red-black tree insertion requires $O(1)$ rotations?
2. Is it true that walking a red-black tree with n nodes in in-order takes $\Theta(n \log n)$ time?
3. Given a red-black tree with n elements, how fast can you sort them using the tree?
4. How fast can we build a red-black tree with n elements?
5. If a data structure supports an operation FOO such that a sequence of n FOO's takes $O(n \log n)$ time in the worst case, then the amortized time of a FOO operation is $\Theta(\quad)$ while the actual time of a single FOO operation could be as low as $\Theta(\quad)$ and as high as $\Theta(\quad)$.

[25 points] **Problem 2:**

In this problem we consider a data structure \mathcal{D} for maintaining a set of integers under the normal INIT, INSERT, DELETE, and FIND operations, as well as a COUNT operation, defined as follows:

- INIT(\mathcal{D}): Create an empty structure \mathcal{D} .
- INSERT(\mathcal{D}, x): Insert x in \mathcal{D} .
- DELETE(\mathcal{D}, x): Delete x from \mathcal{D} .
- FIND(\mathcal{D}, x): Return pointer to x in \mathcal{D} .
- COUNT(\mathcal{D}, x): Return number of elements larger than x in \mathcal{D} .

Describe how to modify a standard red-black tree in order to implement \mathcal{D} such that INIT is supported in $O(1)$ time and INSERT, DELETE, FIND, and COUNT are supported in $O(\log n)$ time.

[40 points] **Problem 3:**

A pharmacist has W pills and n empty bottles. Let $\{p_1, p_2, \dots, p_n\}$ denote the number of pills that each bottle can hold.

a) Describe a greedy algorithm, which, given W and $\{p_1, p_2, \dots, p_n\}$, determines the fewest number of bottles needed to store the pills. Prove that your algorithm is correct (that is, prove that the *first* bottle chosen by your algorithm will be in some optimal solution).

b) How would you modify your algorithm if each bottle also has an associated cost c_i , and you want to minimize the total cost of the bottles used to store all the pills?

Give a recursive formulation of this problem (formula is enough). You do not need to prove correctness.

(Hint: Let $MinPill[i, j]$ be the minimum cost obtainable when storing j pills using bottles among 1 through i . Thinking of the 0-1 KNAPSACK PROBLEM formulation may help.)

c) Describe briefly how you would design an algorithm for it using dynamic programming and analyse its running time.

[25 points] **Problem 4:**

In this problem we look at the amortized cost of insertion in a dynamic table. Initially the size of the table is 1. The cost of insertion is 1 if the table is not full. When an item is inserted into a full table, it first expands the table and then inserts the item in the new table. The expansion is done by allocating a table of size 3 times larger than the old one and copying all the elements of the old table into the new table.

a) What is the cost of the i -th insertion?

b) Using the accounting method, prove that the amortized cost of an insert in a sequence of n inserts starting with an empty table is $O(1)$.

EXTRA CREDIT 10 points

b) Prove the same amortized cost by defining an appropriate potential function. You can use the standard notation $num(T)$ for the number of elements in the table T and $size(T)$ for the total number of slots (maximum size) of the table.