CPS 231 Homework 1

Insertion sort, Mergesort and Growth of Functions

CLRS Chapter 1, 2 and 3

Write and justify your answers on this sheet in the space provided.¹

Exercises (suggested)

1. (CLRS 2.1-2) How do you modify the \textit{INSERTION\textendash\textit{SORT}} procedure to sort into non-increasing instead of non-decreasing order?

2. (CLRS 2.2-4) How can we modify almost any algorithm to have a good best-case running time?

Problems (mandatory)

1. (CLRS 1.2-2) Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size $n$, insertion sort runs in $8n^2$ steps, while merge sort runs in $64n\log n$ steps. For which values of $n$ does insertion sort beat merge sort?

2. (CLRS 1-1) For each function $f(n)$ and time $t$ in the following table, determine the largest size $n$ of a problem that can be solved in time $t$, assuming that the algorithms to solve the problem takes $f(n)$ microseconds.

<table>
<thead>
<tr>
<th></th>
<th>1 second</th>
<th>1 minute</th>
<th>1 day</th>
<th>1 month</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2^n$</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

¹Collaboration is allowed, even encouraged, provided that the names of the collaborators are listed along with the solutions. Write up the solutions on your own.
3. (CLRS 3.1-3) Explain why the statement ‘The running time of algorithm A is at least $O(n^2)$’ is content free.

4. CLRS 2-4 (The inversion problem).
5. (part of CLRS 3-3) Order the following expressions by their asymptotic growth and justify your answer.

\[ 2^n, n!, (\log n)!, n^3, e^n, 2^{\log_2 n}, n \log n, 2^{2^n}, n^{\log \log n} \]