CPS 130 Homework 1 - Solutions

1. (CLRS 1.2-2) Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size n, insertion sort runs in $8n^2$ steps, while merge sort runs in $64n \log n$ steps. For which values of n does insertion sort beat merge sort?

Solution: We want to find n such that $8n^2 \le 64n \lg n$:

$$8n^{2} \leq 64n \lg n \quad \Rightarrow \quad n \leq 8 \lg n$$
$$\Rightarrow \quad \frac{1}{8} \leq \frac{\lg n}{n},$$

which is satisfied for $n \leq 43$.

2. (CLRS 1-1) For each function f(n) and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithms to solve the problem takes f(n) microseconds.

		1 second	1 minute	$1 \mathrm{day}$	$1 \mathrm{month}$
Solution:	n	10^{6}	60×10^6	864×10^8	26×10^{11}
	n^2	10^{3}	7745	29×10^4	16×10^{5}
	2^n	20	26	36	41

3. (CLRS 2.1-2) How do you modify the *INSERTION – SORT* procedure to sort into non-increasing instead of non-decreasing order?

Solution: Modify the while loop test as follows: while i > 0 and A[i] < key