This summer was dedicated to the first stages of the generalization of Professor William Barker's "$L^p$ Harmonic Analysis on $SL(2, \mathbb{R})$" ($0 < p \leq 2$) to the Lie group $SU(2,1)$. The paper consists of two sections, proving two directions of the continuity of the mapping under the generalized operator-valued Fourier transform of the $C^p$ Schwartz space of the group $G = SL(2, \mathbb{R})$ isomorphically to a Schwartz-like space of operators defined on the unitary dual $G^\wedge$ of $G$.

We began with the general structure theory for Lie algebras, understanding and adapting the concepts specifically to $su(2,1)$, the associated Lie algebra of $SU(2,1)$. These topics included (but certainly were not limited to) root space and restricted root space decompositions, Cartan and Iwasawa decompositions, Weyl groups, and universal enveloping algebras. We then moved on to the infinite dimensional representation theory of Lie groups and Lie algebras, developing the Principal and Discrete series representations for $SU(2,1)$. These representations are sufficient for defining the image of the Schwartz space under the Fourier transform. As of the end of the summer research session we have defined the Schwartz space for $SU(2,1)$, parameterized both the discrete and principal series, and understood the $C^2$ case. Further work remains in generalizing the $C^2$ case to the general $C^p$ case, a project which includes the following tasks: finding a common, or relatable, basis for the Discrete and Principal series, adapting the seminorms on the $C^2$ space to the $C^p$ space, decomposing the spherical components of the Schwartz functions into their asymptotic expansions, and understanding the leading exponents of those expansions.

We spent a large portion of our time within the literature on harmonic analysis of semisimple Lie groups. Fortunately this project generalizes in different ways from papers by Peter Trombi, James Arthur, and William Barker (our advisor). Along with (the few) papers published on the abstract theory of $SU(2,1)$, these papers have allowed us to do concrete work in a subject that would normally be beyond an undergraduate level. However, this study would still not have been possible were it not for the patient, energetic, and committed mentoring of our advisor William Barker. We gratefully thank him and the Surdna and Kibbe Foundations for providing this opportunity, and we look forward to continuing our work as Honors projects this upcoming academic year.

Faculty Mentor: William H. Barker, Ph.D.

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