Solving for the Inverse Conformal Factor in Non-Vacuum Environments

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Einstein’s theory of general relativity tells us that the universe is comprised of three spatial dimensions and one time dimension that are woven together to form a four-dimensional “spacetime”. His theory of general relativity states that large amounts of matter or energy will distort or curve the surrounding spacetime. We experience this curvature through the force we call gravity.

We can compute this curvature from Einstein’s equations and we call it the conformal factor; this factor relates the spacetime we’re solving for and a flat spacetime. Black holes, because of their infinite density, cause infinite curvature at their center – and thus an infinite conformal factor. Because Einstein’s equations are non-linear, they must be solved via computers. Computers cannot do computations involving infinity so the conformal factor cannot be solved for at the singularity (the center). To get around this problem, physicists have come up with two alternative ways of solving for the conformal factor in and around a black hole. The excision method simply does not solve for the conformal factor close to the center of the black hole assuming that it will be similar to simple models that may be solved for analytically. The second method, the puncture method, sums an analytical solution to a very basic black hole and a correction factor, which is set to zero at the center of the black hole. While both of these methods are very effective, they are also convoluted and are not as simple as is desired.

This past spring, Professor Baumgarte published a paper[^1] suggesting that we solve instead for the inverse conformal factor, which is just the reciprocal of the conformal factor raised to an arbitrary exponent. Because the inverse of infinity is zero, which is a number the computers can handle, this method allows researchers to solve for the conformal factor without any extra alterations. Not only is this method “cleaner”, it allows us to model some configurations of black holes that we could not model before.

This summer I have been testing this method and comparing it to traditional methods of solving for the conformal factor using code written by Professor Baumgarte this spring. Initially, I was just comparing this method for single black holes in a vacuum. I then modified to code and equations so that they could model configurations that contained matter. I began with a constant density star because there is an analytical solution to which I could compare my data.

In these tests, we were looking to see not just if the inverse conformal method would create accurate results, but also to see how well it solved the equations. These models are solved by comparing data from adjacent points on a grid. For reasons of computational speed, we want to use as few grid points as possible to get accurate data. It is similar to trying to use as few pixels in order to build a recognizable picture. Therefore, even if this method were very accurate, it would need to be accurate on a small grid in order for it to be useful for further calculations. Due to problems in our code, we were unable to determine whether this method was comparable to traditional methods for any matter configurations except for a constant density star.

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References