Einstein’s theory of general relativity is a relativistic theory of gravity. The equations of general relativity, known as Einstein’s equations, describe how matter and energy distort spacetime. Unfortunately, Einstein’s equations are extremely complex. Only a handful of analytical solutions exist, and these solutions require specific symmetries and choices in coordinates. However, we can construct numerical solutions to Einstein’s equations on computers, to either find more general solutions or to explore previous analytical solutions in greater detail.

The Oppenheimer-Snyder solution is one of the few analytical solutions to Einstein’s equations [1]. This solution describes how a spherically symmetric dust cloud with uniform density and zero pressure collapses into a black hole. The purpose of my research project is to examine numerically the same collapse with a different coordinate system and an alternative approach. In particular, I use the BSSN formulation of the 3+1 decomposition equations together with the “moving puncture” coordinate system [2]. This new coordinate system and approach is widely used in numerical relativity for various theoretical experiments. Therefore, it is beneficial to analyze how a well-known solution behaves under these new conditions and verify that everything behaves properly.

Fortunately, Professor J. David Brown has already written code for the BSSN equations [3]. His code, however, is written for a black hole in vacuum. In order to simulate the collapse of a dust cloud, I had to alter the initial data and add hydrodynamic equations that govern the evolution of the dust cloud. With a variety of numerical methods in place, I was able to compile and run my code. Below is a plot of my result (a). Each line corresponds to the density at a different coordinate time. Evidently, the radius decreases and the density increases as expected. If I allow the code to run until the dust cloud’s proper time reaches the analytical collapse time, the density approaches infinity and the radius approaches zero. In other words, the dust cloud has collapsed into a black hole. I also compared the numerical density as a function of proper time to the analytical results derived by Oppenheimer and Snyder. From graph (b), we can see that they essentially match.

Unfortunately, with coding comes numerical error. The numerical methods I use in my code require that my functions are smooth, but there is a discontinuity at the radius. As a result, a numerical error arises, which propagates from the surface inward, which we can see in (c). Graph (c) is of the Hamiltonian Constraint at $t = 0$ and two later time steps. By the design of the BSSN equations, the Hamiltonian Constraint should be zero everywhere at all times. Throughout the year, I will attempt to diminish the numerical error and further explore the collapse in hopes of an Honors project.

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