Perturbations of the Height Function for Kerr Black Holes

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Using the skills I gained from taking General Relativity (Physics 375) this past spring, I examined perturbations of the height function for Kerr black holes in Boyer-Linquist coordinates. The Kerr solution to Einstein’s field equations generalizes the static Schwarzschild solution by including a parameter to account for angular momentum (spin). Just as the Schwarzschild metric describes a static black hole, the Kerr metric describes a rotating black hole. Due to the complexity of the partial differential equations that arise in general relativity, much of the work done involves solving these equations numerically. While computational analysis provides important information about such black holes, analytic solutions often allow for deeper understanding of such systems, and it was in pursuit of such analytic solutions that I spent my time.

I began by examining some of the analytic work done on Schwarzschild black holes, and then chose to look at Kerr black holes in the limit of slow rotation. By examining this limit, I could ignore many terms and greatly simplify the related equations. Additionally, I hoped that similarities would arise between my results and the previous research done on Schwarzschild black holes. However, while initially the Schwarzschild case appeared superficially similar to my own research, I soon realized that the addition of angular momentum brought an additional variable into all of the equations that greatly complicated matters. After making the simplifications to the Kerr metric that arise from assuming slow rotation, I could analytically compute the lapse, shift, and normal vector. Roughly put, these values explain how to measure space and time in the context of general relativity. With these quantities in hand, I then turned to the maximal slicing equation which describes a so-called “maximal” space-time hypersurface.

I devoted the rest of my time to examining this equation and attempting various techniques to break it down into more manageable pieces. Here I first looked at the axisymmetric Schwarzschild case, which includes the additional variable to account for spin, but does not actually allow for the black hole to spin. While the physical possibility of such a situation remains ambiguous, this hypothetical situation furthered my progress. I then proceeded to look at the more complex Kerr case in the slowly rotating limit. Here, I met with mixed results. Working alongside Ken Dennison, I succeeded in breaking one particularly nasty partial differential equations into several much more manageable ordinary differential equations. While we found solutions to several of these equations, one in particular has stumped us and has thus far prevented achieving a satisfying result. I plan to continue researching this subject and searching for a solution into the upcoming school year.

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