Bondi Accretion in Trumpet Geometries

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The accretion of gas onto a black hole is an important and well-studied phenomenon in the field of relativistic astrophysics. As was first demonstrated by the cosmologist and mathematician Hermann Bondi in 1952, this accretion process can be understood analytically under the simplifying assumption of spherical symmetry. Bondi’s original set of equations, referred to collectively as the Bondi solution, has played an important role both in relativistic astrophysics, where it guides our intuition of more general accretion processes, and in numerical relativity, where it serves as a well-understood test case for numerical codes.

The classical Bondi solution is given in so-called Schwarzschild coordinates, the simplest mapping of the Schwarzschild spacetime, or the spacetime containing an isolated and non-rotating black hole. Unfortunately, Schwarzschild coordinates are well suited to describe the exterior of the black hole only, and become singular at the so-called event horizon that encloses the singularity at its center. As a result, most current numerical relativity codes cast black holes in so-called trumpet coordinates, in which slices of constant coordinate time take on a trumpet-like geometry. Unlike Schwarzschild coordinates, trumpet coordinates extend smoothly through the horizon; they also terminate at a finite (nonzero) distance from the center of the black hole, thereby avoiding many of the issues that result from treating the infinite curvature and density at the singularity in a numerical code.

The goal of this project was twofold. First, we aimed to transform the original Bondi solution (in Schwarzschild coordinates) into three different coordinate systems: isotropic Schwarzschild coordinates, maximal trumpet coordinates, and one member of a simple family of coordinate systems that also features a trumpet geometry. We then wanted to use these transformed solutions, together with the original Bondi solution, to construct initial (i.e., input) data for a numerical code. The code would take our initial data and evolve it over time; we would then be able to analyze the evolution of the fluid and compare with expected results. As of this writing, we have completed the first part of the project; work on the second part is ongoing.

Before we could transform the original Bondi solution into our chosen coordinates, however, we first needed to solve Bondi’s equations, i.e., to determine the density and velocity profiles of the accreting fluid. The fluid density is the most difficult to calculate; since there is no analytical expression for the density as a function of radius, it is necessary to solve for it numerically using a root-finding algorithm. Once the density is known, calculation of the fluid velocity is straightforward.

Once we had determined the fluid density and velocity profiles, and then performed the calculations to transform them into the desired coordinates, we could begin to implement these solutions in our numerical relativity code. To this end, we wrote a series of programs (in C++) to solve the fluid equations in each coordinate system. These solutions are then fed into the main code, which evolves them over a specified time. Future work will examine how the initial fluid profiles evolve under different coordinate conditions, and how the results of these simulations compare to previous work.

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