Harmonic Analysis of $SU(2,1)$

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The focus of this summer was the continuation of the work of Justin Dury-Agri ('15) and Andrew Pryhuber ('15), the previous students to undertake this project during last summer and the last academic year. The ultimate goal is to generalize Professor William Barker's “$L^p$ Harmonic Analysis on $SL(2,\mathbb{R})$” to the Lie group $SU(2,1)$. This is a result which we hope can be established in the future for all “real rank one” semisimple Lie groups, of which $SL(2,\mathbb{R})$, $SO(n, 1)$, and $SU(m, 1)$ are examples, for $n$ and $m$ positive integers. Establishing the desired result for these specific cases would offer insight into further methods of proof for the general case. The theorem in question regards the continuity of the generalized operator-valued Fourier transform of the $C^p$ Schwartz space (where $0<p\leq 2$) of a real rank one Lie group $G$ isomorphically to a Schwartz-like space of operators defined on the unitary dual $\hat{G}$ of $G$. The unitary dual of $G$ consists of the irreducible unitary representations of $G$, all but one of which are infinite dimensional.

Much of my time was spent learning the necessary introductory material concerning Lie Theory and Representation Theory, the two main subjects pertaining to the project. This included gaining an understanding of root spaces and restricted root spaces, matrix forms of the adjoint representations of a Lie algebra, as well as the root space, Cartan, and Iwasawa decompositions of the Lie algebras in question. Unlike Dury-Agri's and Pryhuber's analytical focus, I primarily looked at the material from a linear algebra and group theory perspective. Using a basis of matrices for the adjoint representations, I used Mathematica to catalog the decompositions of $SU(2,1)$ and $SO(3,1)$. This catalog summarizes all of the basic structure of the groups and will be a valuable reference for the continuation of this project. In particular, $SO(3,1)$ (not a group considered heavily by Dury-Agri and Pryhuber) should be an easier group to consider than $SU(2,1)$: it has a simpler structure, it is only six dimensional rather than eight dimensional, and has no Discrete Series of representations. This latter condition is a major simplifying condition.

One constraint that prevented me from getting further into the heart of the project this summer was my lack of analysis training (I will not take my first analysis course until this coming fall semester). Analysis is crucial for dealing with the continuity issues of the Fourier transform mapping, as well as analyzing the infinite dimensional representations of our groups. Despite this, my foundational work with the groups, in addition to Dury-Agri's and Pryhuber's analytical work with the Principal and Discrete Series (the only components of the unitary dual that we need to consider), provide me with the opportunity to resume my work during the 2016-17 academic year upon the conclusion of Professor Barker's sabbatical.

Faculty Mentor: William H. Barker, Ph.D.

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