Dynamic Auctions with Asymmetric Information and an Application to Outer Continental Shelf (OCS) Auctions*

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Abstract

The paper studies an asymmetric two-bidder, two-period common-value auction where an informed bidder faces an uninformed one. Both bidders decide whether or not to bid on an object in the first period. If neither bidder places a bid, the informed bidder obtains additional information and the object is re-offered in the second period.

We first consider the situation when the seller is uninformed throughout the game and can set reserve prices: We find that (1) setting different reserve prices for each periods generates more expected revenue than a uniform reserve price for both periods. (2) When the seller is limited to a uniform reserve, it is optimal for the seller to set a low reserve price that allows both bidders to always bid in the first period and guarantees the sale the object. (3) If he can set different reserve prices, the seller optimally sets moderate reserve prices for both periods where the first period reserve price is a slightly higher than the second period reserve price. Bidders no longer always bid in the first period and the probability that the object is sold is less than 1.

We then examine the situation when the seller is initially uninformed but obtains the same information as the informed bidder in the second period of the game and has the option to disclose it: We show that the results and intuition found for an uninformed seller qualitatively carry over to the informed seller. The seller optimally sets a moderate first period reserve price and a low reserve price for the second period.

For both situations we argue that the reserve prices serve two different objectives: To prevent the informed bidder from gaining additional information

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and to elicit more (aggressive) second period bids. Setting a low reserve price in the first period encourages first period bidding and thus the sale of the object before the informed bidder can obtain further information. Setting a high first period reserve discourages first period bidding and conversely encourages second period bidding. Analoguously, a high second period reserve discourages second period bidding but potentially elicits more aggressive second period bids.

Each objective can only be achieved at the expense of the other. When the reserve prices are uniform, preventing the acquisition of information always dominates any increase in second period bidding competition in terms of expected revenue.

If the seller can set different reserve prices for each period, intuition tells us that expected revenue should weakly improve. For uniformly distributed private signals, in fact, we find no improvement in expected revenue from allowing different reserve prices.

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1 Introduction

When auctioning off oil and gas leases on the Outer Continental Shelf (OCS), the federal government frequently offers different leases for sale within the same offshore area every two or three years. In April 2007, Mineral Management Service, the federal agency in charge of managing the mineral resources on federal and Indian lands, completed the last Beaufort Sea lease sale, which was the third of the three sales in the 5-year program for the Beaufort area. The previous two sales had been held in September 2003 and March 2005. During the 2005 lease sale, two blocks in Beechey Point area were for sale. ConocoPhillips and Armstrong Shell already owned neighboring blocks from the 2003 sale and had been doing seismic exploration in the area. They could thus be assumed to be knowledgeable about the value of the blocks for sale. Given their information advantage, one might think they would be interested in bidding on adjacent blocks and prevail in any bidding contest with other less knowledgeable competitors. However, in this instance the North American Civil Recoveries Arbitrage (NACRA Corp.), who owned no blocks in the area and thus had presumably little information about its prospects, ended up winning the leases.

One could presume that ConocoPhillips and Shell had determined that the leases on offer simply were not valuable. However, an alternative explanation is that both expected that no one else would bid on the leases and that they planned to bid on the blocks in the 2007 sale, if they are re-offered. In order to understand why this might be plausible, one needs to bear in mind that oil exploration is a costly endeavor[1] and the companies obtain more private information from exogenous sources (surveys conducted by the federal government or rival companies) as time goes by. The bidder who obtains additional information or has an informational advantage tends to receive more of the surplus. Therefore, a bidder who potentially can obtain additional private information faces a tradeoff between bidding immediately and waiting for more information.

For ConocoPhillips and Armstrong Shell then the alternatives would have been the following: Either submit a bid in the 2005 sale on the new available leases, which is based on existing private seismic survey data and risk the possibility of not finding any deposit; or, decline to bid in the 2005 sale, with the intention to carry out additional surveys or wait for new information on production potential and make a more informed bid in 2007, if the blocks are re-offered. The danger in waiting until 2007 was that in the meantime a rival offer could be made and accepted in the 2005 auction, which would eliminate any future sale.[2]

In the case of aforementioned 2005 OCS auctions, the following questions arise regarding the bidders' strategies: Should the firms which won the previous sale (the

[1]Porter (1995) reported that only 39% of the tracts receiving bids in the period 1954-79 were productive. For the sake of high risks of sinking a dry well and considerable drilling costs, firms often delay drilling and hope for a free ride on other firms' drilling decisions

[2]In addition to the possibility of no bid being made, in OCS auctions, a re-offer can occur because it did not meet an announced minimum bid. Additionally, the federal government can reject the highest bid if they think the bid is too low, based on their private estimate of the value of the tract. A rejected lease is usually reoffered in two to three years.

neighbor firms) wait? Is there any incentive for non-neighbor firms to enter the auction?

From the point of view of the seller, the following questions are of interest: If the government has access to the information obtained by the neighbor firms after the 2005 sale, should it disclose it publicly ? Finally, how should the government set reserve prices for the auctions in order to maximize expected revenue ?

To answer these questions, we introduce an asymmetric two-bidder two-period auction with common values. In such a dynamic setting, waiting increases the informed bidder's information advantage, but also increases the chances of losing the object to a rival bid, which incurs the cost for the informed bidder.

In our model, the seller auctions off an indivisible common-valued object for which there are two bidders, the informed bidder and the uninformed bidder. While the informed bidder receives private information in each of two consecutive periods and the uninformed bidder obtains no private information whatsoever.

Since the informed bidder acquires additional private information in the second period, he faces the trade-off of bidding immediately and waiting for subsequent information. By bidding immediately, he enhances the probability of getting the object, but risks overbidding. Alternatively, if he does not bid in the first period, he risks losing the object to a bid by the uninformed bidder, but he has the chance to be certain about the object's value if the second period is reached. Therefore, the informed bidder must act strategically. His decision, of course, depends on the reserve price and the private information received in the first period.

We find that in such a setting, the informed bidder will bid in the 1st period when the private information in the first period indicates a high value of the object compared to the reserve price, because waiting is costly in the sense that the expected loss from losing the object outweighs the expected benefit of knowing the value of the object for certain. On the other hand, when the private information of the first period is unfavorable compared to the reserve price, the informed bidder has the most to gain from waiting, so he will delay bidding.

We find that when the seller has access to the second period information, he is never worse off by making the second period information public. In specific, we consider two polar opposite policies about information disclosure by the seller: In scenario I, the seller commits to never disclose any information he obtains, while in scenario II, he commits to always disclose it. The intuition follows from the linkage principle by Milgrom and Weber (1982), which states that a seller always benefits from disclosing information that is affiliated to the value of the object. In this case, making the second period information public in a sense subsidizes the uninformed bidder by reducing the information asymmetries between them. It elicits second period bids from the uninformed bidder. This in turn triggers more competitive bids from the informed bidder in the second period and the seller overall benefits from increase in competition in the second period.

Assuming information is uniformly distributed and that the seller sets a uniform reserve price for both periods, we show that in either scenario the seller maximizes expected revenue by setting a low reserve price and the object is sold in the first period. The reason for setting a low reserve price is to discourage the informed bidder from obtaining more (and thus better) information. Since the informed bidder is better informed in the second round, waiting decreases uncertainty so that he better estimates the value of the object, thus he benefits the most from waiting. This implies that from a seller's point of view the less waiting the less rents the informed bidder can extract and the better return for the seller. Setting low reserve prices encourages both bidders to bid in the first period, thus it avoids the informed bidder gaining extra information in the second period. Consequently, it decreases the informed bidder's expected profits and increases the seller's ex ante revenue. This result provides an explanation as to why in OCS auctions the government has been setting low reserve prices.[3]

[3] McAfee and Vincent (1992) suggest that the government has been setting reserve prices consid-

If we allow the seller to set individual reserve prices for each period, we would expect revenue to weakly improve. Given that he can always set both periods' reserve prices equal to the optimal reserve price under a constant reserve price regime, the seller cannot do worse than before. In fact, the seller now has an additional degree of freedom in setting reserve prices and this may be used to extract a higher expected revenue from the auction. The seller may be able to further increase expected revenue by setting a 1st period reserve price that is greater than the 2nd period one. The intuition for this inequality of reserve prices is as follows: The 2nd period reserve price has to be lower in order to provide the uninformed bidder with an incentive to bid in the 2nd period. The seller wants to encourage uninformed bidder to bid in the 2nd period, in order to create more competition in the 2nd period. However, the difference between the two reserve prices must not be too large. By reducing the 2nd period reserve price the seller is giving up some of his surplus in the 2nd round to the bidders.

We show that for signals that are uniformly distributed, the gain from increased competition in the 2nd period is always outweighed by the loss of 2nd period surplus to the seller by reducing the 2nd period reserve price below the 1st period reserve price.

In studying this problem, we draw on both the literature on auctions with asymmetic information and on the literature on auctions with a reoffering feature, where the former relates to the bidders' perspective and the latter to the seller's perspective of the problem.

Previous theoretical and empirical studies on auctions with asymmetric information have mainly focused on the first price, sealed bid auction in a one shot game. Competitive bidding under asymmetric information was first studied by Woods (1965) in a setting of an offshore oil auction. He examined two major oil companies' bidding behaviors and found that the firm, which owned the right of an adjacent tract, bid.

erably lower than the optimal reserve prices in OCS auctions.

The other firm, which had only imperfect seismic information, did not bid. Motivated by his empirical work, Wilson (1967) introduced a theoretical model of a first-price, sealed bid auction. Subsequently, Weverbergh (1979) and Engelbrecht-Wiggans, Milgrom and Weber (1983) (EMW hereafter) furthered theoretical analysis on competitive bidding under asymmetric information. EMW show that the informed bidder's bidding distribution is the same as the distribution of the maximum of the others' bids. The uninformed bidders earn zero expected profit in a one shot game. Following that, Mead, Moseidjord and Sorensen (1984) and Hendricks and Porter (1988) provided empirical evidence to suggest that neighbor firms are better informed than non-neighbor firms. They also suggested that firms bid strategically according to the Bayesian Nash Equilibrium. Furthermore, Milgrom and Weber (1983) extended the model of EMW (1983) by incorporating information acquisition and information disclosure. They showed that the informed bidder prefers to gather information overtly whereas the uninformed bidder prefers to gather information covertly, and that the seller prefers to adopt a policy of disclosing the information that is affiliated with the buyers' valuations.

This paper is also related to the literature on auctions with a reoffering feature. The seller's strategic behavior in such a dynamic auction has been studied by Wang (1993), Horstmann & LaCasse (1997) and McAfee & Vincent (1997). Wang (1993) considers a two-period common-value auction in which the seller possesses some private information at the first period whereas bidders receive some private information at the second period. He shows that, at equilibrium, the seller sets a low reserve price when he receives unfavorable information and sets a high reserve price otherwise. This result is supported whether the seller fully reveals or conceals the information at the beginning of the second period. His results also suggest that the seller benefits from committing the sale in the first period. Unlike this paper, Wang's paper concentrates on symmetrically informed bidders with the reserve price acting as a signaling device. McAfee & Vincent (1997) study a sequential sale of a common value object to symmetrically informed bidders. They show that when the discounting approaches zero, the seller's expected revenue converges to that of a static auction with no reserve price. With many bidders, the equilibrium reserve price approaches the reserve price in an optimal static auction. Similar to Wang (1993) and McAfee & Vincent (1997), Horstmann & LaCasse (1997) also study a common-value auction with symmetrically informed bidders. Offering an explanation for the joint use of secret reserve prices and reoffering, they also predict that as the delay in reoffering increases, so does the average price of items sold.

The paper is organized as follows: Section 2 first introduces the model, followed by section 3, which presents the equilibrium strategies for both scenarios where the government always or never commits to disclosing second period information. Subsequently, in section 4 we investigate the seller's revenue for a simple case where both information signals follow a uniform distribution and the seller sets a uniform reserve price for both periods. First, we compare the seller's revenue between the two scenarios and derive the optimal reserve price. Then, we study the situations if the object is sold only in the first or second period and compare the seller's revenue among different cases. In section 5, we show and discuss how the results are affected if we allow for individual reserve prices for each period. Finally, section 6 concludes.

2 Model

The paper considers the sale of an object by a first price, sealed bid auction with asymmetrically informed bidders I (the informed bidder) and U (the uninformed bidder). The auction opens in two consecutive periods, indexed by $t = 1, 2$. The seller does not observe private signal V_1 in the 1st period, but does observe private signal V_2 at the beginning of the 2nd period. Bidder I observes both V_1 in the first period and V_2 in the second period. Finally, bidder U does not obtain any private information on his own, but may receive information through the seller. We study the setting where the ex-post value of the object is the sum of the two private signals,

$$V = V_1 + V_2$$

where V_1 and V_2 are nonnegative and have finite expectations. We assume both bidders are risk-neutral. There is no discounting in this game.

Initially we assume the seller sets a constant reserve price r for both periods.[4] At the beginning of period 1, each firm independently decides whether to bid in that period or wait until the next period. If any bidder bids in the first period, the bidder with the high bid wins the object, and the auction ends. If both bidders place the same bid then each bidder wins with a probability of $\frac{1}{2}$. If neither firm bids in period 1, the auction opens at the second period, bidder I observes V_2 and each bidder once again decides whether to bid and how much to bid. The auction ends at the end of the second period.

In this model is the sum of information received in both periods by I provides a sufficient estimate for the value of the object. In the application of OCS auctions, if the neighbor firm has drilled the previously purchased tracts, the information revealed from drilling plus the private seismic surveys conducted earlier are considered to be sufficient to predict the deposit potential of adjacent tracts. The assumption that non-neighbor firms have only publicly announced geological data may be somewhat unrealistic, however, as long as the neighbor firms are far more informed than non-neigbor firms, the model is likely to induce similar behavior strategies. Another assumption made in the model is that the reserve price is same in both periods. In OCS auctions, the same winning bid will be rejected again if it were to be rejected in the first period. So the assumption of the same reserve price in both periods appears

[4] In OCS auctions, the government employs a 'secret reserve price' in the sense that the government reserves the right to reject even if the highest bid is above the announced minimum price. The discussion of optimal secret reserve price is beyond the reach of this paper. For theoretical work on secret reserve prices, see Ashenfelter (1989), Elyakime, Laffont, Loisel and Vuong (1994), Horstmann and LaCasse (1993), Wang (1993), Vincent (1995) and working paper by Li and Tan.

to be reasonable.

Additionally, it is assumed that there is only one uninformed bidder and one informed bidder. The number of uninformed bidders is not very important because the equilibrium strategies depend on the distribution function of the maximum of the bids from the uninformed bidders. As for the informed bidders, empirical evidence suggests that they often submit joint bids and reach a unitization agreement (an agreement on how to allocate revenues and costs among firms).

3 Bidders' Equilibrium Strategies

To gain a better understanding of the fundamental characteristics of the bidding behavior, we start by considering a benchmark case of a one period game where one bidder has perfect information about the value of the object while the uninformed bidder does not have any private information. Understanding a one period game helps to derive properties and find equilibrium strategies for a two period game.

3.1 A One Period Game

In a one period game, let $V = V_1 + V_2$ be the value of the object with support $[0, \overline{v}]$ and $F(v)$ be the cumulative probability distribution. An equilibrium of this game is a pair of strategies (β, H) for bidder I and bidder U such that $\beta(v)$ solves bidder I's problem for every realization of v and $H(s)$ maximizes bidder U's expected payoff which is $(E(V|\beta(v) < s) - s)H(s)$ when bidder U bids s. Following EMW (1983), the equilibrium strategies are stated as follows:

Theorem 1 *The Bayesian Nash equilibrium when $E(V|V < \overline{v}) > r$ is:*

- *Bidder I bids according to the strategy* $\beta(v) = \begin{cases} E(V|V < v) & v > \widetilde{v} \\ r & r \leq v \leq \widetilde{v} \\ 0 & v < r \end{cases}$

- *Bidder U chooses a bid from* $\{0\} \cup (r, E(V|V < \overline{v})]$ *according to the cumulative probability distribution* $H(b)$:

$$H(b) = \Pr(b_U \leq b) = \begin{cases} F(\widetilde{v}) & b \leq r \\ F(\beta^{-1}(b)) & b > r \end{cases}$$

where $\widetilde{v}$ *is the value of the signal such that* $E(V|V < \widetilde{v}) = r$.

Theorem 1 extends the theorem by EMW (1983) to asymmetrically informed auctions with a reserve price and is a restatement of the theorem provided by Hendricks and Porter (1988). It states that when the reserve price is lower than the unconditional expected value of the signal V, bidder I bids monotonically according to the signal V and bidder U mixes between bidding and not bidding.

Intuitively, bidder I engages in bid shading where the lower limit of the shaded bid is r. The degree of bid shading has to be such that bidder U becomes indifferent between bidding and not bidding.

More formally, the equilibrium strategy of bidder I on $(r, E(V|V < \overline{v}]$ is determined by the condition that, in equilibrium, bidder U must earn zero expected profits. Since $\beta^{-1}(b)$ is strictly increasing in $b \in (r, E(V|V < \overline{v})]$, there is a unique v such that $v = \beta^{-1}(b)$. Suppose bidder U submits a bid $b \in (r, E(V|V < \overline{v})]$, then his expected payoff if he wins is

$$E(V|\beta(V) < b) - b = E(V|V < \beta^{-1}(b)) - b \tag{1}$$

Setting expression (1) equal to zero yields $\beta(v) = E(V|V < v)$. To induce the aforementioned bidder I's strategies, bidder U has to choose the same distribution function as what bidder I follows above the reserve price. Thus, bidders' bidding distributions are almost identical except for two aspects. Firstly, I's bids range from the reserve price to $E(V|V < \overline{v})$ whereas U always bids strictly above the reserve price. Secondly, the probability of I not bidding, $F(r)$, is less than the probability of U not bidding, $F(\widetilde{v})$. The difference in the probabilities of not bidding arises because I's bidding distribution has a mass point at the reserve price. At equilibrium, I earns positive expected profits while U earns zero expected profits.

Theorem 2 *The Bayesian Nash equilibrium when* $E(V|V < \overline{v}) \leq r$ *is:*

- *Bidder I bids according to the strategy* $\beta(v) = \begin{cases} r & v \geq r \\ 0 & v < r \end{cases}$
- *Bidder U does not bid.*

When $E(V|V < \overline{v}) \leq r$, bidder U will never want to bid above the reserve price since any bid above the reserve price earns him negative expected profits. Suppose bidder U's strategy is to bid r, the optimal response of bidder I is to bid slightly above r if the signal is above that, and not bid otherwise. This implies that bidder U will only win when the expected value is lower than r. Therefore, U loses money by bidding r. As a result, bidder I bids the lowest price and makes positive expected profits and bidder U never bids, as he in essence faces a severe type of winner's curse.

Theorems 1 and 2 show the equilibrium strategies in a one shot game where one bidder has perfect information about the value of the object while the other one does not have any private information. One might wonder about the incentive of the uninformed bidder to bid when the expected value of the object is above r. If the uninformed one does not bid, the optimal response of the informed one is to bid r. But knowing that the informed player bids r, the uninformed player will bid slightly above r. So not bidding cannot be the equilibrium strategy for the uninformed bidder. Meanwhile, it also cannot be the case that the uninformed bidder follows a pure strategy. If the uninformed bidder bids a certain value, the informed bidder will bid slightly above that whenever the signal is higher than the uninformed bidder's bid. So the uninformed bidder will win only when the signal is below his bid and he will take a loss. Therefore, the equilibrium strategy of the uninformed bidder is a mixed strategy.

3.2 A Two Period Game

Now we turn to the discussion of the two period game. We consider to possible scenarios when the seller obtains V_2 at the beginning of the 2nd period: Scenario I, when the seller commits to keep the information secret and scenario II, when the seller commits to disclosing this information to bidder U. We first derive some general properties for the equilibrium strategies in both scenarios, then we focus on the equlibrium strategies in each scenario respectively. Define F_i as the cumulative probability of V_i with support $[0, \overline{v}_i]$. To ensure "well behaved" equilibria, we make the following assumptions:

Assumption 1. The two signals V_1 and V_2 are statistically independent.[5]

Assumption 2. For each $i \in \{1, 2\}$, the distribution F_i has no atom in its support.[6]

For the convenience of notation, let $E_i = E(V_i)$ and $E_{v_i} = E(V_i | V_i < v_i)$.

Lemma 3 *Bidder U earns zero profit and plays mixed strategies whenever he bids.*

The intuition for this lemma follows closely the argumentation used in the one period game. Since bidder U has no private information in the first period, in any Nash equilibrium in which he plays mixed strategies in the first period, he must earn zero profit if he wins in the first period. Suppose bidder U makes positive expected profits in the first period, then he will never want to wait until the next period and the game becomes a one shot game. However, as mentioned in the last section, the expected profits of the uninformed bidder can only be zero in a one shot game. Therefore, bidder U earns zero profits in both periods.

[5] The assumption that the signals received by the informed bidder are independent is made to obtain a clean characterization of the equilibrium and its properties. The behavior implication would be the same were it assumed that the signals are affiliated, as long as the degree of affiliation is 'weak' and the informed bidder still has the incentive to wait for subsequent information.

[6] According to the results obtained by Engelbrecht-Wiggans (1981), assuming atomless distributions does not have a qualitative effect on the equilibrium.

Given that the first period signal follows a atomless distribution, we can easily write down bidder I's pure strategies. A pure strategy for bidder I in the first period is given by a function β_1 which maps the set of first period signals into a decision space {B, N} where B represents that he bids in the first period, and N represents that he does not bid in the first period. In case of bidding in the first period, bidder I has to participate in such a manner that bidder U earns zero profit if he wins. Bidder I's bidding strategies in the first period are summarized in the following lemma.

Lemma 4 *Suppose bidder I bids in the first round, in equilibrium, he bids* $\beta_1(v_1) = E_{v_1} + E_2$ *when* $E_{v_1} + E_2 > r$ *, and bids* $\beta_1(v_1) = r$ *when* $E_{v_1} + E_2 \leq r$*.*

Proof. The proof follows the same logic as the one in the one-period setting. By bidding $b \in (r, E_1+E_2]$ in the first period, bidder U's payoff if he wins is $E(V|\beta_1(V_1) < b) - b$. Since $\beta_1^{-1}(b)$ is strictly increasing in $b \in (r, E_1 + E_2]$, there is a unique v_1 such that $v_1 = \beta_1^{-1}(b)$. Thus,

$$\begin{aligned} E(V|\beta_1(V_1) < b) - b &= E(V|V_1 < \beta_1^{-1}(b)) - b \\ &= E(V_1|V_1 < \beta_1^{-1}(b)) + E(V_2|V_2 < \overline{v}_2) - b \\ &= E_{\beta_1^{-1}(b)} + E_2 - b \end{aligned}$$

By lemma 3, $E(V|\beta_1(V_1) < b) - b = 0$. Then we have

$$E_{\beta_1^{-1}(b)} + E_2 - b = 0 \Leftrightarrow \beta_1(\beta_1^{-1}(b)) = b = E_{\beta_1^{-1}(b)} + E_2$$

Substituting $\beta_1^{-1}(b)$ with v_1 yields $\beta_1(v_1) = E_{v_1} + E_2$. Thus, we have $\beta_1(v_1) = E_{v_1} + E_2$ when $E_{v_1} + E_2 > r$. Since $\beta_1(v_1)$ cannot be less than r, $\beta_1(v_1) = r$ when $E_{v_1} + E_2 \leq r$. ■

Let B_I and B_U denote the support of bids for bidder I and U in the first period. Then bidder U's bidding strategies in the first period are summarized in the following lemma.

Lemma 5 *Suppose bidder U bids in the first period, in equilibrium,* $B_I \supseteq B_U$*. When he bids above r, he follows the same bidding distribution as what bidder I follows, which is* $F_1(v_1)$*.*

Again, the rationale is same as the one in the one period game. In order to induce the above mentioned bidder I's strategies, bidder U has to choose a bidding distribution that mirros bidder I's bidding behavior.

Suppose that neither of the player bids in the first period and that the auction moves to the second period. Since there is no future period, the second period auction is a one period game. Since the outcome of the first period contains information, after the first period bidder U updates his prior on the basis of no bidding from bidder I in the first period. Therefore, the equilibrium strategies differ from those in the described one period game in that both bidders follow the distribution of the value of the object conditional on player I not having bid in the first period. Let $\widetilde{F}(v)$, defined on the support $[v^L, v^H] \subseteq [0, \overline{v}]$, be the probability distribution that updates the prior $F(v)$ after bidder U knowing bidder I did not bid in the first period.[7] The equilibrium bidding strategies in the second period are given in the following lemma.

Lemma 6 *Suppose the auction moves to the second period, players' bidding strategies follow those in the described one period game, except that their bids follow the bidding distribution* $\widetilde{F}(v)$*.*

Lemma 6 implies that the information conveyed from the first period affects bidder's bidding behavior only through the bidding distribution. This is because bidder U remains less informed than bidder I and the updated beliefs incorporate all the information available to him.

3.2.1 Scenario I: Bidder U remains uniformed in 2nd period

Given a reserve price and the first period signal v_1, bidder I decides whether or not to bid in the first period. Let $\pi_1(v_1)$ be the expected payoff if he bids in the first period

[7]In the scenario where the seller commits to disclosing v_2, v_2 in a sense becomes public information. Thus, bidders' updated prior is not only based on no-bidding in the first period but also based on v_2, and it is formed such that $\widetilde{f}(v) = \widetilde{f}(v|v_2)$.

and $\pi_2(v_1)$ be the expected payoff if he waits until the second period. Knowing bidders' bidding behaviors described above, we have

Lemma 7 *$\pi_1(v_1)$ is increasing in v_1.*

Proof. By lemma 4 and 5, $\pi_1(v_1)$ is either

$$(v_1 - E_{v_1})F_1(v_1) \tag{2}$$

if $E_{v_1} + E_2 > r$ or

$$(v_1 + E_2 - r)(\gamma + \frac{p}{2}) \tag{3}$$

if $E_{v_1} + E_2 \leq r$ and $v_1 \geq r - E_2$ where γ and p denote bidder U's probability of not bidding and bidding r. It can be shown easily that both (2) and (3) are increasing in v_1. ■

Lemma 8 *$\pi_2(v_1)$ is increasing in v_1.*

Proof. By lemma 6, $\pi_2(v_1)$ is written as

$$\gamma \int_{\max(0,\, r-v_1)}^{\max(\overline{v}_2,\, r-v_1)} (v - \beta_2(v))\widetilde{F}(v)dF_2(v_2) \tag{4}$$

The expression (4) is the expected payoff in the described one-shot game with $\widetilde{F}(v)$ as bidder U's belief, weighted by the probability that bidder U does not bid in the first period. An increase in v_1 represents a higher value on v. Since $\widetilde{F}(v)$ is a updated prior of $F(v)$ and the expected payoff in a one shot game $(v-\beta_2(v))F(v)$ is increasing in v, it must be case that $(v - \beta_2(v))\widetilde{F}(v)$ is increasing in v_1 as well. Then, it follows immediately that $\pi_2(v_1)$ is increasing in v_1. ■

When the auction moves to the second period, bidders update their priors, taking into account that no bidding occured in the first period. As a consequence, the updated prior $\widetilde{F}(v)$ has less weight on higher values of $v = v_1 + v_2$. Furthermore, both $\pi_1(v_1)$ and $\pi_2(v_1)$ are increasing functions of v_1, where $\pi_1(v_1)$ increases faster in v_1 than $\pi_2(v_1)$. Therefore, it is natural to look for equilibria in which bidder I bids in the first period when v_1 is above a cut-off value and waits till the second period otherwise.

Let

$$V_1^* = \{v_1^* \in [0, \overline{v}_1] : \pi_1(v_1^*) = \pi_2(v_1^*)\}$$

be the set of threshold values at which bidder I is indifferent between bidding and waiting, given the belief that above which bidder I bids in the first round and below which bidder I waits. If for all $v_1 \in [0, \overline{v}_1]$, $\pi_1(v_1) > \pi_2(v_1)$, then $v_1^* = 0$. If for all $v_1 \in [0, \overline{v}_1]$, $\pi_1(v_1) < \pi_2(v_1)$, then $v_1^* = \overline{v}_1$. In equilibrium, we have

Proposition 9 *1. when $r \in [0,\ E_2]$, $v_1^* = 0$;*

2. when $r \in (E_2, E_1 + E_2)$, v_1^* satifies $E(V_1|V_1 < v_1^*) + E(V_2|V_2 < \overline{v}_2) = r$;

3. when $r \in [E_1 + E_2, \overline{v}_1 + \overline{v}_2]$, $v_1^* = \overline{v}_1$.

The key aspect in this two period game is the trade-off between bidding immediately and waiting for additional information by the informed bidder. By bidding immediately, bidder I enhances the probability of winning the object but risks misinterpreting the value of the object. When the reserve price is lower than the expected value of the second period signal, the advantage of obtaining the object is more important than the advantage of knowing the value of the object for certain and bidder I will always bid in the first round. When the reserve price is higher than the expected value of object, bidder U will never bid because of the potential for a negative payoff. Obviously, the advantage of waiting for more information dominates over the advantage of bidding in the first period for bidder I, thus in this case bidder I will always wait. When the reserve price is moderate, the dominant advantage depends on the signal received in the first period. As the value of the first period signal increases, waiting for subsequent information becomes more costly and the benefit of obtaining the object becomes more important. So bidder I bids when the signal is high and waits when the signal is low.

Given that bidder U bids randomly in the first period, his bidding behavior is represented by a bidding distribution. In equilibrium, we have

Proposition 10 *Bidder U's bidding distribution in the first period is define by*

$$H_1(b) = \begin{cases} \gamma & b < r \\ \gamma + p & b = r \\ F_1(\beta_1^{-1}(b)) & b > r \end{cases}$$

where γ and p satisfies the following equations

$$\gamma + p = F_1(v_1^*)$$
$$(\gamma + \tfrac{p}{2})(v_1^* + E_2 - r) = \gamma \int_{\max(0,\ r-v_1^*)}^{\max(\overline{v}_2,\ r-v_1^*)} (v_1^* + v_2 - r) dF_2(v_2).$$

The left hand side of the second equation is bidder I's expected payoff at v_1^* if he bids in the first period.[8] The right hand side of the second equation is bidder I's expected payoff at v_1^* if he waits till the second period. The solution to this two period problem has the feature that bidder I decides to bid (wait) whenever the first period signal exceeds (is lower than) v_1^*. When the first period signal is equal to v_1^*, he is indifferent between bidding and waiting. The characterization of bidders' equlibrium strategies in this two period game is summarized in the following theorem.

Theorem 11 *The Perfect Bayesian Nash equilibrium in the two period game is: At $t = 1$, bidder I bids $E_{v_1} + E_2$ if $v_1 \in [v_1^*, \overline{v}_1]$ and does not bid otherwise; bidder U randomizes his decision on the support*

$$B_U = \begin{cases} [E_2, E_1 + E_2] & \text{when } r \in [0,\ E_2] \\ \{0\} \cup [r, E_1 + E_2] & \text{when } r \in (E_2, \overline{v}_1 + \overline{v}_2] \end{cases}$$

according to the c.d.f. $H_1(b)$. At $t = 2$, bidder I bids r whenever $v > r$ and does not bid otherwise; bidder U does not bid at all.

[8] Assume each bidder gets the object with the probability equal to $\frac{1}{2}$ when there is a tie at the reserve price.

Theorem 11 extends the results of an asymmetric information game to a dynamic setting. It states that bidder I's behavior depends on the values of both the reserve price and the first period signal to decide whether to bid or to wait until the second period, while bidder U randomizes his bid if he bids in the first round and never bids in the second round.

If the reserve price is low, $r \leq E_2$, bidder U always bids in the first period and the object is sold with certainty. This is now possible, because unlike the previously described one period game, bidder I's expected value of the object is always larger than the reserve price, thus the reserve price serves no role.

If $r > E_2$ bidder U will decline to bid in the first period and the auction will proceed to the 2nd period with some positive probability. Since the expected value of the object, conditional on $v_1 < v_1^*$, is no larger than the reserve price in this case, it follows according to Theorem 2, that bidder U will not bid. To sum up the rationale for bidder U's bidding strategy: He does not have anything to gain by waiting till the second period. So he either bids in the first round or does not bid at all.

In OCS oil and gas lease auctions, the federal government will reject the high bid if the bid is considered to be too low. Hendrick et al. (1988) report that the government regularly rejects winning bids and reoffers it in the future. Their statistics show that the bids received on reoffered leases are generally lower than those sold in previous offerings. This is consistent with our result that when the auction goes to the second period the only bid received is the reserve price, whereas the winning bid can be anything between the reserve price and the expected value of the object if the auction ends in the first round.

3.2.2 Scenario II: Bidder U learns of V_2 at the beginning of the 2nd period

Similar to the first scenario, we investigate the equilibrium strategies in which the informed bidder bids in the first round if the signal is high and waits till the second round if the signal is low.

Let

$$V_1^{**} = \{v_1^{**} \in [0, \overline{v}_1] : \pi_1(v_1^{**}) = \pi_2(v_1^{**})\}$$

be the set of threshold values at which bidder I is indifferent between bidding and waiting, given the belief that above which bidder I bids in the first round and below which bidder I waits. If for all $v_1 \in [0, \overline{v}_1]$, the expected profit from bidding in the first period exceeds that of bidding in the second period, such that $\pi_1(v_1) > \pi_2(v_1)$, then this implies $v_1^{**} = 0$, or that all types of bidder I bid immediately. Conversely, if for all v_1, it holds that $\pi_1(v_1) < \pi_2(v_1)$, then this implies $v_1^{**} = \overline{v}_1$. In equilibrium, we have

Proposition 12 *1. when $r \in [0,\ E_2]$, $v_1^{**} = 0$;*

2. when $r \in (E_2, E_1 + E_2)$, v_1^{**} satifies

$$v_1^{**} + E_2 - r = \int_{\max[r - v_1^{**}, 0]}^{\min[r - E_{v_1^{**}}, \overline{v}_2]} (v_1^{**} + v_2 - r) \quad dF_2(v_2) + \int_{\min[r - E_{v_1^{**}}, \overline{v}_2]}^{\overline{v}_2} (v_1^{**} - E_{v_1^{**}}) \quad dF_2(v_2); \tag{5}$$

and $v_1^{**} < v_1^{*}$.

3. when $r \in [E_1 + E_2, \overline{v}_1 + \overline{v}_2]$, $v_1^{**} = \overline{v}_1$, because for all $v_1 \in [0, \overline{v}_1]$,

$$v_1 + E_2 - r < \int_{\max[r - v_1, 0]}^{\min[r - E_{v_1^{**}}, \overline{v}_2]} (v_1 + v_2 - r) \quad dF_2(v_2) + \int_{\min[r - E_{v_1^{**}}, \overline{v}_2]}^{\overline{v}_2} (v_1 - E_{v_1}) \quad dF_2(v_2)$$

, .

In this scenario, we also distinguish three cases for bidders' equilibrium strategies. When the reserve price is lower than E_2, bidder I always bids in the first round and the object is always sold. Since the settings in both scenarios differ only in the second round, the bidding strategies must be the same if the auction ends in the first round. Therefore, the equilibrium strategies are indeed identical in both scenarios when $r < E_2$. However, this is no longer the case when the reserve price is higher than E_2. The threshold value v_1^{**} may be lower than v_1^*, the one in scenario I, which implies that, as compared to scenario I, bidder I bids more frequently in the first period. The intuition is that disclosure of the second period information induces the uninformed bidder to bid in the second round, thus it triggers a competition in the second round. The competition elicits aggressive bids from the informed bidder which in turn decreases the rents he can extract in the second period. Therefore, the informed bidder tends to bid more frequently in the first round and waits less.

In this scenario, since bidder U observes v_2 in the second period, v_2 becomes a public information. Hence, the information settings in two periods are similar in the sense that v_1 is the only private information bidder I has which bidder U does not have. When the auction moves to the second period, bidder U updates his prior based on no-bidding in the first period. Consequently, bidder U's bidding distribution is given by a conditional probability distribution, $F_1(\beta_2^{-1}(b)|V_1 < v_1^{**})$. The equilibrium strategies in the second period are summarized in the following theorem.

Theorem 13 *For any Perfect Bayesian Nash equilibrium at $t = 2$, there exists a threshold value $\widetilde{v_1} \in [0, v_1^{**}]$ above which bidder I bids $E_{v_1} + v_2$, below which bidder I either bids r or does not bid. If the value of v_2 is sufficiently high, bidder U may bid in the second period and his bid follows the c.d.f. $H_2(b) = F_1(\beta_2^{-1}(b)|V_1 < v_1^{**})$ whenever $b > r$.*

Theorem 13 shows the equilibrium strategies in the second round for the scenario when the seller commits to disclosing v_2. The bidding strategies resemble the equilibrium outcome presented in the described one period game, that is, bidder I bids monotonically based on the values of signal v_1 and v_2, and bidder U randomizes his bid whenever he participates. The major difference between two scenarios is that bidder U may bid in the second period if v_2 is high enough, whereas he never participates in the second round in scenario I. This is because bidder U can gain information by waiting in this scenario whereas he has nothing to gain by waiting in scenario I. As a consequence, we may expect bidding from both bidders in the second round.

The important consequence of Theorem 13 is that, as we will show later, making the second period information public decreases information asymmetries between two bidders in the second round, thus inducing competition which in turn decreases the rents the informed bidder can extract and increases the seller's revenue.

Proposition 14 *Bidder U's bidding distribution in the first period is given by*

$$H_1(b) = \begin{cases} F_1(v_1^*) & b \leq r \\ F_1(\beta_1^{-1}(b)) & b > r \end{cases}$$

Theorem 15 *The Perfect Bayesian Nash equilibrium at $t = 1$ is: Bidder I bids*

$$\beta_1(v_1) = \begin{cases} E_{v_1} + E_2 & \text{when } v_1 \in [v_1^*, \overline{v}_1] \\ r & \text{when } v_1 \in [v_1^{**}, v_1^*] \\ 0 & \text{otherwise.} \end{cases}$$

Bidder U randomizes his decision on the support

$$B_U = \begin{cases} [E_2, E_1 + E_2] & \text{when } r \in [0, \ E_2] \\ \{0\} \cup (r, E_1 + E_2] & \text{when } r \in (E_2, \overline{v}_1 + \overline{v}_2] \end{cases}$$

according to the c.d.f. $H_1(b)$.

Proposition 14 and Theorem 15 present the equilibrium strategies in the first period when the seller commits to disclose v_2 in the second round. Similar to Theorem 11, bidder I bids when the value of the signal is high and waits when the value of the signal is low, while bidder U randmizes his decision on whether to bid and how much to bid. Together with Proposition 12, Theorem 15 suggests that, when $r \in (E_2, E_1 + E_2)$[9], bidder I possesses a mass point at the reserve price. This is exactly opposite to the results in scenario I, in which bidder U possesses a mass point at the reserve price when $r \in (E_2, E_1 + E_2)$.

[9] This is the situation when $v_1^{**} \neq v_1^*$.

3.2.3 Equilibrium Strategy Comparisons

In this section, we focus on the differences and similarities of the equilibrium strategies in scenario I and II. In the table below, we show the probability of bidding when $r \in (E_2, E_1 + E_2)$ for bidder $i \in (I, U)$ and scenario $j \in (N, D)$ where N and D represent scenario I and scenario II respectively.

	I	U
N	$1 - F_1(v_1^*)$	$1 - F_1(v_1^*) + p$
D	$1 - F_1(v_1^{**})$	$1 - F_1(v_1^*)$

Let p_i^j denote the probability of bidding in the first period when $r \in (E_2, E_1 + E_2)$ for bidder i and scenario j. One then has the following:

Lemma 16 *Since $v_1^{**} \leq v_1^*$, we have*

1. $p_I^N < p_U^N$, $p_I^D > p_U^D$;
2. $p_I^N < p_I^D$, $p_U^N > p_U^D$.

The results from the comparisons are summarized as follows: First, bidder I bids more frequently than bidder U in the first round whereas the situation is exactly opposite in the first scenario. Second, compared with the first scenario, the likelihood of bidding in the first round is higher for bidder I but lower for bidder U in the second scenario.

The equilibrium differences between two scenarios imply two distinct effects of allowing the seller to commit to disclosing v_2, the 'frequent bidding effect' and the 'immediate bidding effect.' The frequent bidding effect measures bidder I's probability of bidding in the first round versus bidder U's probability of bidding in the first round, and it is shown by the first two inequalities. The frequent bidding effect favors bidder I in the sense that bidding in the first round becomes more attractive for bidder I but less attractive for bidder U when the seller commits to disclosing v_2, thus bidder I bids more frequently than bidder U in the first round. The immediate bidding effect measures bidders' probabilities of bidding immediately versus their probabilities of waiting, and it is shown by the second two inequalities. Since bidder I's information advantage is lessened in the second round when the seller commits to disclosing v_2, he has less incentive to wait thus tends to bid immediately. Conversely, since bidder U can obtain information by waiting, he has more incentive to wait thus tends not to bid immediately. In other words, committing to disclosing v_2 decreases the degree of information asymmetry so that, in contrast to the analysis in the first scenario, waiting may become beneficial for the uninformed bidder but not for the informed one.

Corollary 17 *In both scenarios, when $r \in [0,\, E_2]$, bidders' strategies are exactly the same; when $r \in (E_2, E_1 + E_2)$, bidder U may bid in the first period while bidder I does not.*

When the reserve price is lower than E_2, both bidders bid in the first round and the object is always sold. Since the settings in both scenarios differ only in the second round, the bidding strategies must be the same if the auction ends in the first round. Indeed, the equilibrium strategies are identical in both scenarios when $r < E_2$. When $r \in (E_2, E_1 + E_2)$, in both scenarios, bidder I follows the strategy of bidding when the signal is high and waiting when the signal is low, while bidder U randomizes his decision on whether to bid and how much to bid. This establishes an interesting situation where we may observe bidding from the uninformed bidder instead of the informed one in the first around. In fact, this phenomenon is observed in many economic environments. For example, we often observe situations of entering into a new product market by a new firm instead of an incumbent. The firm which enters into the market later has to bear the possibility of being the follower and earning a smaller market share, but gains more accurate information on the new market. This result provides an understanding for why such situations exist, and suggest when we may observe it as well.

4 Seller's Revenue Analysis

The issue addressed in this section is whether the seller, in order to maximize the ex ante expected revenue, should commit to disclosing the second period information when he chooses freely the reserve price. In order to simplify the analysis, we assume both signals follow a uniform distribution in $[0, 1]$.[10] I first compare the revenue in two scenarios and show that the seller weakly prefers to commit to disclosing the second period information. Then we turn to the optimal reserve price and show that, at low reserve prices, the seller maximizes the expected revenue by setting the reserve price lower than the expected value of the second signal in both scenario. Furthermore, we analyze how results are affected if the seller can commit beforehand to sell the object in one of the two periods. Our main result is that committing to selling the object in a specific period does not change the maximized ex ante expected revenue received in the two period game and the optimal reserve prices will remain low enough as to guarantee a sale. This seems to indicate that the feature of selling the object more than once does not reduce the seller's expected revenue at all.[11]

4.1 Revenue analysis in a two Period Game

We first discuss the ex ante probabilities of trade. Our objective is to compare the ex ante probability of trade when the seller commits to not disclosing the second period information, p_1, with the ex ante probability of trade when the seller commits to disclosing the second period information, p_2. Under our assumption of signals following a uniform distribution in [0, 1], the threshold value for bidder I in the first scenario, v_1^*, is $2r - 1$ and the threshold value for bidder I in the second scenario, v_1^{**},

[10] The results would be the same if both signals follow a uniform distribution in $[a, b]$ since the normalization of the signals only has quantitative effect on bidders' payoff.

[11] Wang (1993) obtained the opposite result in terms of the seller's ability to sell on more than one occasion. He shows that if the seller can somehow commit himself to selling in the first period, he will receive more revenue.

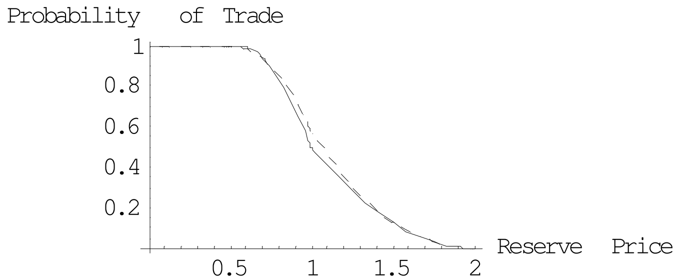

Figure 1: Committing to disclosing v_2 versus committing to not disclosing v_2.

is $\frac{2(2r-1)}{3}$. One then has:

$$p_1 = \begin{cases} 1 & \text{when } r \in [0, \frac{1}{2}] \\ (2-2r) + (2-2r+p)(2r-1) + \gamma \int_0^{v_1^*} \int_{\min(r-v_1,1)}^1 dv_2 dv_1 & \text{when } r \in (\frac{1}{2}, 1) \\ \int_0^1 \int_{\min(1,r-v_1)}^1 dv_2 dv_1 & \text{when } r \in [1, 2] \end{cases}$$

$$p_2 = \begin{cases} 1 & \text{when } r \in [0, \frac{1}{2}] \\ (2-2r) + (1-v_1^{**})(2r-1) + (2r-1)v_1^{**}B & \text{when } r \in (\frac{1}{2}, 1) \\ (1-v_1^{**}) + v_1^{**}B & \text{when } r \in [1, 2] \end{cases}$$

where $B = \int_{\min(1,r-\frac{v_1^{**}}{2})}^{\min(r,1)} (1 - \frac{r-v_2}{v_1^{**}} \frac{2r-2v_2}{v_1^{**}}) dv_2 + \int_{\min(1,r)}^1 dv_2 + \int_{\min(1,r-v_1^{**})}^{\min(1,r-\frac{v_1^{**}}{2})} (1 - \frac{r-v_2}{v_1^{**}}) dv_2$

After comparing p_1 with p_2, one has the following result.

Proposition 18 *When the seller has access to v_2, the probability of trade is never decreased by committing to disclosing v_2. For $r \in [\frac{1}{2}, \frac{3}{2}]$, $p_2 > p_1$.*

In Figure 1, the solid line represents the first scenario where the seller commits to not disclosing v_2 and the dashed one represents the second scenario where the seller commits to disclosing v_2. One sees that for $r \in [0, \frac{1}{2}]$ and $r \in [\frac{3}{2}, 2]$ the seller is indifferent between committing to disclosure or committing to nondisclosure, while for $r \in [\frac{1}{2}, \frac{3}{2}]$, the seller strictly prefers to commit to disclosing v_2. For $r \in [0, \frac{1}{2}]$, bidding behaviors are identical in both scenarios, thus the corresponding probabilities of trade are the same as well. For $r \in [\frac{3}{2}, 2]$, in both scenarios, bidder U declines to bid since the expected value of the object to bidder U can never be above $\frac{3}{2}$ before or after v_2 is disclosed. Bidder I then faces a "take it or leave it" situation and two scenarios become the same as well. For $r \in [\frac{1}{2}, \frac{3}{2}]$, committing to disclosing v_2 increases the amount of trade compared to the case when the sller coommits to not disclosing v_2, by triggering the competition in the second period. Disclosing the second period information provides bidder U incentive to participate in the second period, thus the object can be sold to both bidders.

Figure 1 also reflects that in equilibrium, the probability of trade is a decreasing function of the reserve price in both scenarios. It is strictly decreasing over the reserve price starting at $\frac{1}{2}$ and the decreasing rate is increasing for $r \in (\frac{1}{2}, 1)$ and decreasing for $r \in [1, 2]$. This result provides a key technical tool to analyze the change of the seller's revenue corresponding to the reserve price.

We now turn to the discussion of the seller's ex ante revenue. Let R_1 be the ex ante revenue in the case of committing to not disclosing the second period information. We find that depending on the reserve price r, in the 1st period (i) both bidders always bid, (ii) bid some of the time, or (iii) never bid (though bidder I may bid in the 2nd period). To be precise:

$$R_1 = \begin{cases} (i)\ 2\int_0^1 \frac{1+v_1}{2} v_1 dv_1 = \frac{5}{6} & \text{when } r \in [0, \frac{1}{2}] \\ (ii)\ 2\int_{v_1^*}^1 \frac{1+v_1}{2} v_1 dv_1 + r(2r-1)p + \gamma \int_0^{v_1^*} \int_{\min(r-v_1,1)}^1 r dv_2 dv_1 & \text{when } r \in (\frac{1}{2}, 1) \\ (iii)\ \int_0^1 \int_{\min(1,r-v_1)}^1 r dv_2 dv_1 & \text{when } r \in [1, 2] \end{cases}$$

Similarly, let R_2 be the ex ante revenue in the case of committing to disclosing the second period information. Again we can distinguish situations where (i) both bidders always bid, (ii) bid some of the time, or (iii) never bid in the 1st period. It follows that:

$$R_2 = \begin{cases} (i)\ 2\int_0^1 \frac{1+v_1}{2} v_1 dv_1 = \frac{5}{6} & \text{when } r \in [0, \frac{1}{2}] \\ (ii)\ 2\int_{2r-1}^1 \frac{1+v_1}{2} v_1 dv_1 + r(2r-1)\int_{v_1^{**}}^{2r-1} dv_1 + (2r-1)\int_0^{v_1^{**}} A dv_1 & \text{when } r \in (\frac{1}{2}, 1) \\ (iii)\ \int_{v_1^{**}}^1 r dv_1 + \int_0^{v_1^{**}} A dv_1 & \text{when } r \in [1, 2] \end{cases}$$

where $A = 2\int_{\min(1, r-\frac{v_1}{2})}^1 (\frac{v_1}{2} + v_2)\frac{v_1}{v_1^{**}} dv_2 + \int_{\min(1,\max(r-\frac{v_1^{**}}{2}, r-v_1))}^{\min(1, r-\frac{v_1}{2})} r\frac{2r-2v_2}{v_1^{**}} dv_2$
$+ \int_{\min(1,r-v_1)}^{\min(1,\max(r-\frac{v_1^{**}}{2}, r-v_1))} r dv_2$

Note that A represents the expected revenue when the auction does continue to the 2nd period. The possible outcomes in such a case depend on the 2nd period signal v_2: If v_2 is high, bidder I bids above the reserve price in the 2nd period. If v_2 is low, bidder I bids the reserve price and - depending on how low v_2 is - he will or will not face bidding competition from bidder U.

After comparing R_1 with R_2, one has the following result.

Proposition 19 *When the seller has access to v_2, he weakly prefers to commit to disclosing v_2. For $r \in [\frac{1}{2}, \frac{3}{2}]$, the seller strictly prefers to commit to disclosing v_2.*

Like in Figure 1, the solid line represents the first scenario where the seller commits to not disclosing v_2 and the dashed one represents the second scenario where the seller commits to disclosing v_2. The logic follows closely the one for the probability of trade, and the intuition is based on 'Linkage Principle' by Milgrom and Weber (1982) which states that a seller always benefits from disclosing information that is affiliated to the value of the object. When the reserve price is moderate, publicizing the second period information in a sense favors the uninformed bidder by reducing information asymmtries between them. It elicits bidding from the uninformed bidder, thus avoids the loss due to lack of competition in the second round. As a result, the informed bidder fails to fully exploit his information advantage and bids more aggressively,

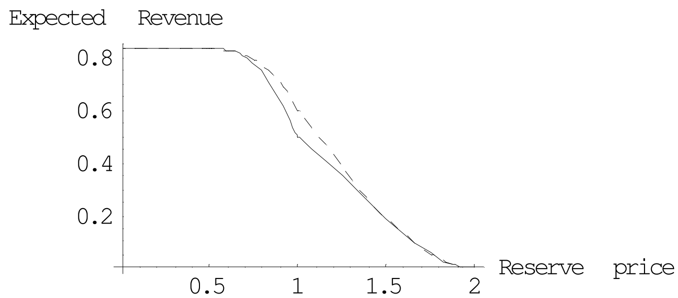

Figure 2: Committing to disclosing v_2 versus committing to not disclosing v_2.

which implies a lower expected profit to the informed bidder but a higher return to the seller. Note that the seller's expected revenue in the first scenario is not differentiable at $r = 1$. This is explained by the fact that when bidder U does not participate in the auction the seller's revenue is expressed by a different function.

Similar to the change of the probabilities of trade, the ex ante expected revenue is also a decreasing function of the reserve price in both scenarios. Hence, one has the following result:

Proposition 20 *In both scenario, the seller maximizes the ex ante expected revenue by setting* $r \in [0, \frac{1}{2}]$.

When $r \in [0, \frac{1}{2}]$, both bidders always bid in the first round and the object is always sold. When the reserve price is above $\frac{1}{2}$, the object may not be sold and any increase in the reserve price lowers the probability that bidders place a bid. Although a higher payment occurs from an increase in the reserve price, the gain from a higher payment is always smaller than the loss from a lower probability of trade. After all, any increase in the reserve price results in a decrease in the seller's revenue. Intuitively, since the informed bidder is the only one which has the private information, setting low reserve prices encourages both bidders to bid in the first

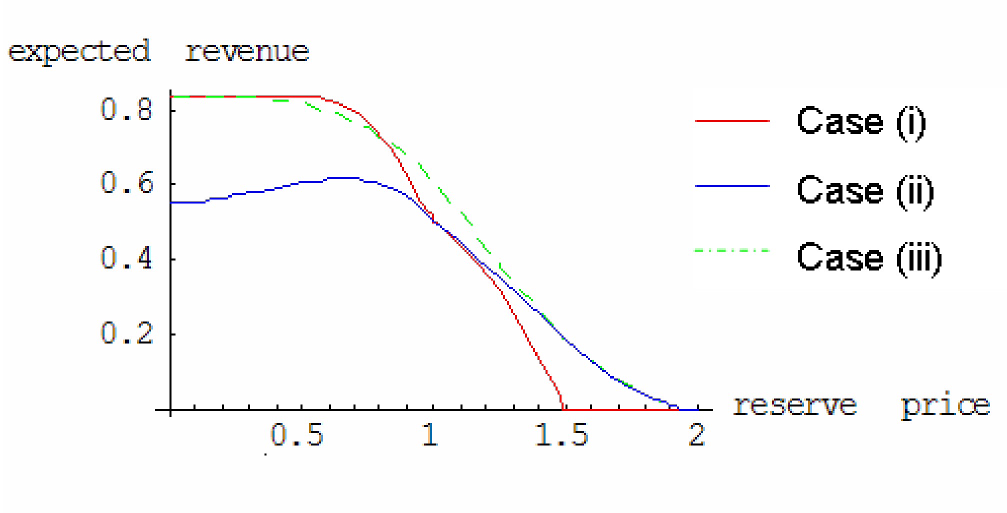

Figure 3: One period game revenue comparison

period, thus this decreases the informed bidder's information advantage by negating his gaining more information in the second period. Consequently, it lowers the informed bidder's expected profit and raises the seller's expected revenue.

The result implies that, at a optimal reserve price, the auction ends in the first period. One may wonder that to what extent the results would change if the seller can commit to selling the object in only one period. In the next section, we investigate the situations where the object is sold either in the first or second round and compare the seller's ex ante revenue under different cases.

4.2 Revenue analysis in a one period game

It is assumed that the seller sets the reserve price before he observes any information. In the following graphs (the expressions for the seller's revenue are provided in the appendix), case (i) is the situation when the object is sold in the first period; case (ii) is the situation when the object is sold in the second period and the seller commits to not disclosing v_2; case (iii) is the situation when the object is sold in the second period and the seller commits to disclosing v_2. The findings are summarized as follows.

Proposition 21 *When the seller has the option to conduct the auction in only one period, he will either sell the object in the first period and set $r \in [0, \frac{1}{2}]$, or sell the object in the second period with zero reserve price and commit to disclosing the second period. In both cases, the ex ante revenue is maximized, and the maximized ex ante revenue is the same as the one in the two period game, which is equal to $\frac{5}{6}$.*

In case (i), the reasoning about the seller's revenue remains unchanged from the two period game. For $r \in [0, \frac{1}{2})$, both bidders always participate and their bids are always above the reserve price. Thus, the seller's expected revenue is the same as in the two period game and it is equal to $\frac{5}{6}$. For $r \in [\frac{1}{2}, \frac{3}{2}]$, the seller's revenue changes with the payment and the probability of bidding. As the reserve price goes up, the payment goes up, yet the probability of bidding goes down. Again, the seller does not benefit from the increase of the reserve price, since the probability of bidding goes down at a more rapid rate. In case (iii), the expected revenue is calculated before v_2 is observed by the seller, and is shown as decreasing over the reserve price. The reason is essentially the same as in case (i) when $r \in [\frac{1}{2}, \frac{3}{2}]$. Although the availability of v_2 in case (iii) affects the ends of the bidding support, the shape of the bidding distribution remains the same as in case (i) since v_1 is the only private information bidder I has which bidder U does not have in both cases. Consequently, the seller's revenue is maximized at $r = 0$ in both case (i) and (iii), and their maximized ex ante revenue is $\frac{5}{6}$ which is the same as the one in the two period game. In contrast, in case (ii) the private information bidder I has includes both v_1 and v_2. Consequently, the bidding distribution becomes concentrated in the middle. This indeed changes the trade-off faced by the seller as the reserve price goes up. As a result, the expected revenue is maximized at $r = 0.65$ and the maximized revenue is lower than $\frac{5}{6}$.

When comparing these three cases, we also observe that for the reserve prices between $\frac{1}{2}$ and 1, the expected revenue in case (i) changes at a more rapid rate than in both case (ii) and (iii). This is due to the fact that an increase in the reserve price is more effective in reducing the expected revenue in the first period. In the first period, a higher reserve price leads to a reduction in the probability of bidding immediately. While in the second period the probability of bidding is affected not only by an increase of the reserve price, but also by the value of v_2. Therefore, the effect of the reserve price is dampened by the presence of v_2. In other words, the reserve price becomes less effective in decreasing the probability of bidding in the second period, thus the decrease of the probability only partially reflects the increase of the reserve price.

An important implication from this result is that committing to selling the object in only one period does not change the surplus generated to the seller, since seller's optimal strategy is indeed to sell in the first period even with the ability to sell on more occasions. This result provides an explanation to why the U.S. government has been setting the reserve prices substantially low and maintaining the reoffering feature in OCS auctions.[12]

Proposition 22 *Let R_I, R_{II}^N and R_{II}^D be the expected revenue for case (i), (ii) and (iii). Then, we have*

1. $R_{II}^N \leq R_{II}^D$ *for all* $r \in [0, 2]$.
2. $R_I \geq R_{II}^N$ *for* $r \in [0, 1]$ *and* $R_I < R_{II}^N$ *for* $r \in (1, 2]$.
3. $R_I \geq R_{II}^D$ *for* $r \in [0, 0.8]$ *and* $R_I < R_{II}^N$ *for* $r \in (0.8, 2]$.

The proposition shows that, as expected, if the object is sold only in the second period, the seller is never worse off by making the second period information public. The intuition follows from the linkage principle by Milgrom and Weber (1982), which states that a seller always benefits from disclosing information that is affiliated to the value of the object. When comparing R_I with R_{II}^N, at first glance, it seems that

[12] McAfee and Vincent (1992) considers a one-period common-value model and shows that the U.S. government has been setting the reserve price substantially lower than the optimal reserve prices in OCS auctions.

R_I should always be higher. Since the informed bidder is better informed about the object's value in case (ii), the uninformed bidder is more vulnerable (than in case (i)) to the winner's curse (winning because the informed bidder holds a lower estimated value of the object), and as a result he shades his bid even more in case (ii). After all, we may expect a higher expected profit for the informed bidder and a smaller R_{II}^N. However, this is only true when the reserve price is lower than the expected value of the object which is one. The reason is that there exists another effect. In case (i), the informed bidder is uncertain about the value of the object because he is only partly informed, and this uncertainty becomes intensified as the reserve price goes up. Thus, when the reserve price is higher than the expected value of the object, this 'uncertainty effect' induces the informed bidder to bid more cautiously. As a consequence, the informed bidder bids less often in case (i) than in case (ii) which in turn leads to a lower R_I. Indeed, the similar rationale carries over to the revenue comparison between case (i) and case (iii). That is, there also exists a threshold value below which R_I is higher, above which R_{II}^D is higher. Since R_{II}^D is higher than R_{II}^N, the threshold value here is less than the threshold value between case (i) and case (ii). In general, as the the reserve price increases, selling the object only in the second period is more likely to benefit the seller.

5 Allowing different reserve prices in each period

We now allow the seller to set a different reserve price for each period. Let r_1 be the reserve price for the 1st period and r_2 be the reserve price for the 2nd period. We will assume that the seller announces the reserve prices at the beginning of the first period. In all other aspects we retain the assumptions of original model.

First, consider how the change in setting the reserve prices will affect bidding under scenario I (when the seller commits not to disclose v_2). As before, bidder I has to decide whether to bid in the 1st period. He will do so if expected profit from bidding in the 1st period, $\pi_1(v_1)$, is greater than the expected profit if he waits until the second period, $\pi_2(v_1)$. Analoguous to the prior scenario, let the 1st period expected profit be

$$\pi_1(v_1) = (v_1 + E(V_2) - E(V|V < v_1 + v_2))\ F_1(v_1), \text{ if } E(V|V < v_1 + v_2) > r_1$$
$$= (v_1 - E(V_2) - r_1)(\gamma + \frac{p}{2}), \text{ otherwise}$$

where γ denotes bidder U's probability of not bidding and p the probability of bidding the reserve price r_1 and $E(V|V < v_1 + v_2)$ and r_1 represent the possible bids by bidder I.

Let the expected profit from the 2nd period bidding be

$$\pi_2(v_1) = \gamma \int_{Max(0,r_2-v_1)}^{Min(1,r_2-v_1)} (v_1 + v_2 - \beta(v_1, v_2))\ \widetilde{F}(\beta_2^{-1}(b))\ dv_2$$

where $\beta_2(v_1, v_2)$ is bidder I's 2nd period bidding function, $\beta_2^{-1}(b)$ is the corresponding inverse bidding function and $\widetilde{F}(.) = F(.|V_1 < v_1^*)$.

We know that if we set $r_1 \leq \frac{1}{2}$ we can replicate the equilibrium outcome for scenario I with constant reserve prices. Thus, the expected revenue cannot be below $\frac{5}{6}$.

However, it is not immediately clear whether an increased 1st period reserve price can improve expected revenue. We do know that a reserve price $r_1 > 1$ will fail to improve expected revenue, because it discourages first-period bidding entirely and effectively turns the auction into a single-period auction. From our previous discussion of single-period auction we know that they are weakly dominated in terms of expected revenue by two-period auctions.

We therefore focus on $r_1 \in (\frac{1}{2}, 1]$, where bidders exhibit the following bidding behavior in equilibrium :

Corollary 23 *If* $r_1 \in (\frac{1}{2}, 1]$, *in an equilibrium bidder I bids* $Min[\frac{v_1}{2} + \frac{1}{2}, r_1]$ *if* $v_1 \geq v_1^*$ *and waits till the 2nd round if* $v_1 < v_1^*$, *where* $v_1^* = 2r_1 - 1$.

In the first round, bidder U randomly chooses from $\{0\} \cup [r, 1]$ *according to the distribution*

$$H_2(b) = \Pr(b_2 \leq b) = \begin{cases} \gamma & b = 0 \\ \gamma + p & b = r_1 \\ 2b - 1 & b > r_1 \end{cases}$$

γ *and* p *represent bidder U's probability of not bidding and bidding* r_1 *respectively, and they satisfy*

$$\gamma + p = 2r_1 - 1$$

$$(\gamma + \tfrac{p}{2})(v_1^* + \tfrac{1}{2} - r_1) =$$
$$\gamma\{\int_{\min(v^* - v_1^*, 1)}^{1} (v_1^* + v_2 - E(V|V < v_1^* + v_2))\ \widetilde{F}(v_1^* + v_2)\ dv_2$$
$$+ \int_{\max(r_2 - v_1^*, 0)}^{\min(v^* - v_1^*, 1)} (v_1^* + v_2 - r_2)\ \widetilde{F}(v^*)\ dv_2\}$$

In the second round, bidder I bids

$$\beta_2(v_1, v_2) = \begin{cases} E(V|V < v) & v > v^* \\ r_2 & r_2 \leq v \leq v^* \\ 0 & v < r_2 \end{cases}$$

and bidder U bids randomly from $\{0\} \cup (r_2, E(V|V < v)]$ *according to the distribution* $H_2(b) = \begin{cases} \widetilde{F}(v^*) & b \leq r_2 \\ \widetilde{F}(\beta_2^{-1}(b)) & b > r_2 \end{cases}$ *where* $E(V|V < v^*) = r_2$.

v_1^* defines the marginal type for bidder I that is indifferent between bidding in the 1st and 2nd period and v^* defines the highest type of bidder U that is bidding the reserve price r_2 in the 2nd period.

The expected revenue to the seller is given by the sum of payments he can expect from the 1st period,

$$2\int_{v_1^*}^{1} \beta_1(v_1)F(v_1)dv_1 + r_1\ v_1^*\ p$$

when either both bidders follow the 1st period bidding function $\beta_1(v_1)$ or when bidder I declines to bid, and from the 2nd period,

$$\gamma\int_0^{v_1^*} A\ dv_1$$

where $A = 2\int_{Max[0,v^*-v_1]}^{1} \beta_2(v_1,v_2)\ \widetilde{F}(v)dv_2 + \int_{Max[0,r_2-v_1]}^{Max[0,v^*-v_1]} r_2\ \widetilde{F}(v^*)dv_2$, which distinguishes between the situations in which both bidders place bids and only bidder I places a 2nd period bid.

The expected revenue thus is:

$$E = 2\int_{2r_1-1}^{1} \frac{1+v_1}{2} v_1 dv_1 + r_1(2r_1-1)\ p + \gamma\int_0^{v_1^*} A\ dv_1$$

$$\text{where } A = 2\int_{Max[0,v^*-v_1]}^{1} E(V|V<v)\ \widetilde{F}(v)\ dv_2 + r_2\int_{Max[0,r_2-v_1]}^{Max[0,v^*-v_1]} r_2\ \widetilde{F}(v^*)\ dv_2$$

Using numerical methods, we find in terms of expected revenue to the seller that the highest expected revenue fails to improve on the maximized revenue of scenario I. We thus conclude:

Proposition 24 *If the seller can set variable reserve prices, he maximizes expected revenue by setting $r_1 \leq \frac{1}{2}$ and r_2 arbitrarily. His expected revenue will be equal to the maximized expected revenue under scenario I with constant reserve prices.*

6 Conclusion

In this paper we have explored bidders' incentives to wait in a first price, sealed bid auction where one bidder has private information in two consecutive periods and the other has only public information. The problem is examined in two situations where the seller has access to the second period information received by the informed bidder: he either commits to not disclosing the second period information or commits to disclosing it. We have shown that the informed bidder delays bidding more frequently in the first situation while the uninformed one does the exact opposite. The rationale is that publicizing the second period information diminishes the informed bidder's information advantage in the second round, while providing the uninformed bidder with an incentive to wait.

We have shown that in both situations, the ex ante revenue is maximized by setting low reserve prices, if a the reserve price for both periods have to be the same. In such a situation the informed bidder is better informed in the second round and thus waiting decreases uncertainty. By delaying his bid the informed bidder takes less risk than bidding in the first round, thus he benefits most from waiting. Consequently, the seller prefers immediate bidding, thus he sets low reserve prices. If we allow for different reserve prices for each period, the seller preference for 1st round bidding is now tempered by the ability to induce bidding competition in the 2nd round.

In addition to the two-period model, we have studied the situation when the seller has the option to sell the object in only one period. We have found that if the auction is conducted in the first round, setting low reserve prices is optimal for the seller. If, on the other hand, the auction is conducted in the second round, then setting zero reserve price and committing to disclosing the second period information generates most surplus to the seller. Nevertheless, the maximized ex ante revenue of the one period game is the same as the one in the two period game.

Our findings suggest an explanation for why in practice we may observe bidding from a uninformed bidder instead of an informed one. They also provide insights on how the government should manipulate the information possessed by the firms to affect the chances of their participation and the amount of revenue generated in OCS auctions.

Appendix

Proof of Theorem 1. Adopting the arguments by EMW (1983) and Krishna (2002), one can show that the above strategies are an equilibrium outcome:

Suppose bidder U employs a mixed strategy $H(b)$. Consider bidder I's bidding strategy $\beta(z)$ where he chooses z to maximize expected payoff. Hence he maximizes:

$$(v - \beta(z))H(\beta(z)) \tag{6}$$

Let $F(z) = H(\beta(z))$, such that

$$(v - \beta(z))F(z) \tag{7}$$

We now show that bidder I's bidding strategy $\beta(z)$ when bidder I's type is v is a direct revelation mechanism such that $b = \beta(v)$. Differentiating (7) with respect to z yields the first-order condition

$$f(z)v - \frac{d}{dz}(F(z)\beta(z)) = 0 \tag{8}$$

Solving the differential equation (8) we find $\beta(z) = \frac{\int_r^z tf(t)dt}{F(z)}$. Notice that for all $z \in [\widetilde{v}, \overline{v}]$, both $\beta(z)$ and $\frac{\int_r^z tf(t)dt}{F(z)}$ are contained in $[r, E(V|V < \overline{v}]$ and monotonically increasing in z. If we substitute $\beta(z)F(z)$ with $\int_r^z tf(t)dt$ in equation (8) then we obtain

$$f(z)(v - z) \tag{9}$$

The expression (9) is non-negative for $z < v$ and non-positive for $z > v$. Thus, the optimal response is to choose $z = v$ when $v \in [\widetilde{v}, \overline{v}]$ and bidder I's bidding strategy is indeed the direct revelation mechanism $b = \beta(v)$. When $v \in [r, \widetilde{v}]$, (9) is non-positive for all $z \in [\widetilde{v}, \overline{v}]$, so bidder I's optimal response is to bid $\beta(\widetilde{v}) = r$.

We know turn to bidder U: Suppose bidder I's follows bidding strategy $\beta(v)$. Given $b \in (r, E(V|V < \overline{v}]$, bidder U's expected payoff if he wins is

$$E(V|\beta(V) < b) - b \text{ or } E(V|V < \beta^{-1}(b)) - b$$

We know from above that $\beta(v) = \frac{\int_r^v tf(t)dt}{F(v)} = E(V|V < v)$. Thus, $E(V|V < \beta^{-1}(b)) = \beta(\beta^{-1}(b))$ and the expected payoff becomes

$$\beta(\beta^{-1}(b)) - b = 0$$

Furthermore if bidder U does not bid his expected payoff is also 0. Thus, bidder U's expected payoff is always 0 and therefore he is willing to employ a mixed strategy $H(b)$, where he randomly chooses a bid from $(r, E(V|V < \overline{v}]$ and sometimes does not bid at all. The reason that bidder U cannot bid r in equilibrium is as follows: Suppose bidder U bids r and wins, then his expected payoff is

$$E(V|\beta(V) \leq r) - r = E(V|V \leq \beta^{-1}(r)) - r$$

Let $\mathbf{v}$=$\beta^{-1}(r)$, the value of the signal received when bidder I bids r. Since bidder I has a mass point at r, $\mathbf{v} \in [r, \widetilde{v}]$. Therefore, we have

$$E_{\mathbf{v}}[E(V|V \leq \mathbf{v})] - r \leq E(V|V \leq \widetilde{v}) - r = 0$$

Therefore, U will never bid r as part of his mixed strategy $H(b)$.

The proof of being the unique equilibrium is essentially the same as the one given by EMW (1983).

Proof of Lemma 5. Let $\underline{b}$ and $\bar{b}$ be the minimal and maximal element of B_I respectively. Consider a bid b by bidder U, if $b <\underline{b}$, bidder U will lose for certain; if $b > \bar{b}$, bidder U can be better off by bid slightly lower than b. If B_I is continuous, we have $B_I \supseteq B_U$ obviously. If B_I is not continuous, so that for some $\tilde{b} \in [\underline{b}, \bar{b}]$, $\tilde{b} \notin B_I$. Then for all $\tilde{b}$, bidder U is better off bidding $b = \max\{ b|b < \tilde{b}$ and $b \in B_I \}$. Thus, we have $B_I \supseteq B_U$ as well. The proof of bidder U's bidding distribution above r follows closely the one in the one period game. ■ ■

Proof of Propositions 9, 10 and Theorem 11. Suppose there exists a threshold v_1^* above which bidder I bids and below which bidder I waits. If $v_1 > v_1^*$, according to Lemma 4, bidder I bids $\beta_1(v_1) = E_{v_1} + E_2$ when $E_{v_1} + E_2 > r$ and bids r otherwise.

Following the same arguments as in Theorem 1, in order to induce such strategies from bidder I in the first round, bidder U has to follow the same distribution above the reserve price. Thus, the equilibrium strategies above the reserve price in the first round follow. If the auction moves to the second round, the game becomes a one shot game. According to Theorem 1 and 2, the equilibrium strategies in the second round can be verified easily too. Hence, what remains to be proven is the threshold value for any given reserve price and the equilibrium strategies at the threshold value.

We first look at the situation given under item 2 of proposition 9. Suppose the threshold $\widehat{v}_1 > v_1^*$, according to Lemma 4, the support of bidder I's bids in the first round is $[E_{\widehat{v}_1} + E_2, E_1 + E_2]$. Given $E(V_1|V_1 < v_1^*) + E(V_2|V_2 < \overline{v}_2) = r$ and $\widehat{v}_1 > v_1^*$, we have $E_{\widehat{v}_1} + E_2 > r$. So bidder U's expected payoff in the first period by bidding r is

$$\begin{aligned} E(V|\beta_1(V_1) < r) - r &= E(V|V_1 < \widehat{v}_1) - r \\ &= E(V_1|V_1 < \widehat{v}_1) + E(V_2|V_2 < \overline{v}_2) - r > 0 \end{aligned}$$

According to Theorem 2, we know that the uninformed bidder cannot earn a positive expected payoff. Therefore, the threshold cannot be above v_1^*.

Now suppose there exists a threshold $\widehat{v}_1 < v_1^*$, by Lemma 4, the support of bidder I's bids in the first round will be $[r, E_1 + E_2]$. Since

$$E(V|V < \overline{v}) = E(V_1|V_1 < \widehat{v}_1) + E(V_2|V_2 < \overline{v}_2) < r,$$

by Theorem 2 bidder I will bid r when $v_1 + v_2 \geq r$ and U does not bid at the second period. So for $v_1 \in (\widehat{v}_1, v_1^*)$, if bidder I decides to wait, his expected payoff is

$$\gamma' \int_{\max(0,\, r-v_1)}^{\max(\overline{v}_2,\, r-v_1)} (v_1 + v_2 - r) dF_2(v_2) \tag{10}$$

(γ' represents the probability that bidder U does not bid in the first round.); if he decides to bid in the first round, he will bid r. Note that bidder U cannot bid r in the first round because he earns negative profit when $v_1 \in (\widehat{v}_1, v_1^*)$. So the probability that I will win by bidding r is γ' and his expected payoff by bidding r in the first round is

$$\gamma'(v_1 + E_2 - r) \tag{11}$$

For $v_1 \in (\widehat{v}_1, v_1^*)$, (10) $\geq$ (11) which implies bidder I will not bid in the first round when $v_1 \in (\widehat{v}_1, v_1^*)$. Therefore, $\widehat{v}_1$ cannot be the equilibrium threshold and v_1^* is the only possible threshold existed at equilibrium for case 2.

Now we will show v_1^* is indeed the threshold value for bidder I and also prove the equilibrium strategies at the reserve price in this case.

First, suppose bidder U does not possess a mass point at the reserve price in the first round. Let γ represent U's probability of not bidding in the first round. At v_1^*, bidder I's expected payoff if he bids in the first round is $\gamma(v_1+E_2-r)$, while if bidder I waits until the second round, his expected payoff is $\gamma \int_{\max(0,\, r-v_1)}^{\max(\overline{v}_2,\, r-v_1)} (v_1+v_2-r)dF_2(v_2)$. Since (10) $\geq$ (11), $\gamma(v_1+E_2-r) \leq \gamma \int_{\max(0,\, r-v_1)}^{\max(\overline{v}_2,\, r-v_1)} (v_1+v_2-r)dF_2(v_2)$ for all v_1. Since bidder I has to be indifferent between bidding in the first round and waiting till the second round at v_1^*, this can not be the equilibrium strategies.

Now consider the case when bidder U possesses a mass point at the reserve price in the first round. In this case, bidder I bids the reserve price only at v_1^* and unlike bidder U thus has no mass point at the reserve price. If bidder U bids the reserve price, his expected payoff if he wins is

$$E(V|V_1 < v_1^*) - r = E(V_1|V_1 < v_1^*) + E(V_2|V_2 < \overline{v}_2) - r = 0.$$

Let p and γ be the probabilities of bidding r and not bidding in the first round by bidder U. Assume each bidder gets the object with the probability equal to $\frac{1}{2}$ when there is a tie. Then, bidder I's expected payoff if he bids in the first round turns into $(\gamma + \frac{p}{2})(v_1^* + E_2 - r)$, while the expected payoff if he waits till the second round remains the same. Since $\gamma(v_1 + E_2 - r) \leq \gamma \int_{\max(0,\, r-v_1)}^{\max(\overline{v}_2,\, r-v_1)} (v_1 + v_2 - r)dF_2(v_2)$ for all

v_1, it follows that there must exist an unique pair of values for γ and p such that

$$\gamma + p = F_1(v_1^*)$$
$$(\gamma + \tfrac{p}{2})(v_1^* + E_2 - r) = \gamma \int_{\max(0,\, r-v_1^*)}^{\max(\overline{v}_2,\, r-v_1^*)} (v_1^* + v_2 - r)dF_2(v_2).$$

Hence, v_1^* can be the threshold value for bidder I and the result follows.

We now turn to item 1 and 3 of proposition 9: Under item 1, suppose $v_1^* > 0$, according to Lemma 4, bidder I always bids above the reserve price. So bidder U's expected payoff in the first period by bidding r is

$$E(V) - r \quad = E_1 + E_2 - r.$$

Since $r \leq E_2, E(V) - r > 0$. Bidder U will always bid the reserve price, so I's equilibrium strategies in the first round can not hold. Thus, v_1^* must be 0. In Under item 3, bidder U never participates. So if bidder I bids in the first round he bids the reserve price and the expected payoff is

$$v_1 + E_2 - r;$$

if he waits till the second round his expected payoff is

$$\int_{\max(0,\, r-v_1)}^{\max(\overline{v}_2,\, r-v_1)} (v_1 + v_2 - r)dF_2(v_2).$$

The rest of the proof is exactly the same as the one in case 2 and the result follows.

■

Proof of Theorem 15. The proof follows similar steps as the proof of Theorem 11. If bidders bid in the first round, according to Lemma 4, bidder I bids $E_{v_1} + E_2$ when $E_{v_1} + E_2 > r$ and bids r otherwise; according to Theorem 1, bidder U follows the same distribution except possible at r and 0. If the auction moves to the second round, bidders bid according to the strategies provided in Theorem 13. Thus, the crucial step is to prove the threshold value.

We start with case 2. Firstly, v_1^{**} can not be greater than v_1^*. The proof of that is exactly the same as the one given in the proof of Theorem 11. Hence, if there exists a threshold, it must be that $v_1^{**} \in [r - E_2, v_1^*]$. At the threshold, if bidder I bids in the first round, his expected payoff is

$$v_1^{**} + E_2 - r; \tag{12}$$

if he waits till the second round, his expected payoff is

$$\int_{\max(0,r-v_1^{**})}^{r-E_{v_1^{**}}} (v_1^{**} + v_2 - r)dF_2(v_2) + \int_{r-E_{v_1^{**}}}^{\bar{v}_2} (v_1^{**} - E_{v_1^{**}})dF_2(v_2). \tag{13}$$

Let $D = (12) - (13)$. Suppose $v_1^{**} = r - E_2$, then,

$$\begin{aligned} D &= - \int_{\max(0,r-v_1^{**})}^{r-E_{v_1^{**}}} (v_1^{**} + v_2 - r)dF_2(v_2) - \int_{r-E_{v_1^{**}}}^{\bar{v}_2} (v_1^{**} - E_{v_1^{**}})dF_2(v_2) \\ &= - \int_{E_2}^{r-E_{v_1^{**}}} (v_2 - E_2)dF_2(v_2) - \int_{r-E_{v_1^{**}}}^{\bar{v}_2} (r - E_2 - E_{v_1^{**}})dF_2(v_2). \end{aligned}$$

By definition, $E_2 + E_{v_1^*} = r$, thus $r - E_2 - E_{v_1^*} = 0$. Since $E_{v_1^*} \geq E_{v_1^{**}}$, one has $r - E_2 - E_{v_1^{**}} \geq 0$. So the second term of the above expression is nonpositive. Since the first term of the above expression is nonpositive obviously, we have $D \leq 0$. Now suppose $v_1^{**} = v_1^*$, then,

$$\begin{aligned} D &= v_1^* + E_2 - r - \int_{\max(0,r-v_1^*)}^{r-Ev_1^*} (v_1^* + v_2 - r)dF_2(v_2) - \int_{r-E_{v_1^*}}^{\bar{v}_2} (v_1^* - E_{v_1^*})dF_2(v_2) \\ &\geq v_1^* + E_2 - r - \int_{\max(0,r-v_1^*)}^{r-Ev_1^*} (v_1^* - E_{v_1^*})dF_2(v_2) - \int_{r-Ev_1^*}^{\bar{v}_2} (v_1^* - E_{v_1^*})dF_2(v_2) \\ &= v_1^* - E_{v_1^*} - \int_{\max(0,r-v_1^*)}^{\bar{v}_2} (v_1^* - E_{v_1^*})dF_2(v_2) \\ &= (v_1^* - E_{v_1^*})F_2(\max(0, r - v_1^*)) \\ &\geq 0. \end{aligned}$$

Thus, we have $D \geq 0$. Also, one can easily show that $\frac{\partial D}{\partial v_1^{**}} > 0$. Hence, there must exist an unique $v_1^{**} \in [r - E_2, v_1^*]$ such that $D = 0$ which implies the result in case 2. The proof for case 3 is essentially the same as the one for case 2. The only difference is that $v_1^{**} \in [r - E_2, 1]$ instead since I never bids above the reserve price.

At $v_1^{**} = r - E_2$, $D \leq 0$ still holds. However, at $v_1^{**} = 1$, one might see $D \leq 0$. This implies that bidder I is always better off by waiting till the second round, thus we have $v_1^{**} = 1$ under some situations. In case 1, since $r \leq E_2$, $v_1^{**} \in [0, 1]$. At v_1^{**}, bidder I's expected payoff if he bids in the first round is $v_1^{**} - E_{v_1^{**}}$, if he waits till the second round his expected payoff is the same as expression (13). Thus, the difference of the expected payoff between bidding and waiting is

$$D = v_1^{**} - E_{v_1^{**}} - \int_{\max(0, r - v_1^{**})}^{r - Ev_1^{**}} (v_1^{**} + v_2 - r) dF_2(v_2) - \int_{r - E_{v_1^{**}}}^{\bar{v}_2} (v_1^{**} - E_{v_1^{**}}) dF_2(v_2).$$

Since $E_2 + E_{v_1^{**}} \geq r$, it follows that

$$\begin{aligned} D &= v_1^{**} - E_{v_1^{**}} - \int_{\max(0, r - v_1^{**})}^{r - Ev_1^{**}} (v_1^{**} + v_2 - r) dF_2(v_2) - \int_{r - E_{v_1^{**}}}^{\bar{v}_2} (v_1^{**} - E_{v_1^{**}}) dF_2(v_2) \\ &\geq v_1^{**} - E_{v_1^{**}} - \int_{\max(0, r - v_1^{**})}^{r - Ev_1^{**}} (v_1^{**} - E_{v_1^{**}}) dF_2(v_2) - \int_{r - E_{v_1^{**}}}^{\bar{v}_2} (v_1^{**} - E_{v_1^{**}}) dF_2(v_2) \\ &\geq 0. \end{aligned}$$

This implies that bidding in the first round is always more favorable for I than waiting till the second round. Thus, $v_1^{**} = 0$ and the result follows. ■

The following lemmas are provided for Proposition 21 and 22.

Lemma 25 1. *When* $r \in [0, \frac{1}{2}]$, $R_I = 2\int_0^1 \frac{1+v_1}{2} v_1 dv_1$.

2. *When* $r \in (\frac{1}{2}, 1)$, $R_I = 2\int_{2r-1}^1 \frac{1+v_1}{2} v_1 dv_1 + \int_{r-\frac{1}{2}}^{2r-1} r(2r-1) dv_1$.

3. *When* $r \in [1, \frac{3}{2}]$, $R_I = r(\frac{3}{2} - r)$.

Lemma 26 1. *When* $r \in [0, \frac{1}{3}]$, $R_{II}^N = 2\int_{v^*}^2 (v^2 - \frac{v^3}{3} - \frac{1}{3}) \frac{2-(2-v)^2}{2} d\frac{2-(2-v)^2}{2} + r\frac{2-(2-v^*)^2}{2}[\frac{2-(2-v^*)^2}{2} - \frac{r^2}{2}]$

where v^* *satisfies* $v^{*2} - \frac{v^{*3}}{3} - \frac{1}{3} = r$.

2. *When* $r \in (\frac{1}{3}, 1)$, $R_{II}^N = 2\int_{v^*}^1 \frac{v^3}{3} \frac{v^2}{2} d\frac{v^2}{2} + 2\int_1^2 (v^2 - \frac{v^3}{3} - \frac{1}{3}) \frac{2-(2-v)^2}{2} d\frac{2-(2-v)^2}{2} + r\frac{v^{*2}}{2}(\frac{v^{*2}}{2} - \frac{r^2}{2})$

where $v^* = (3r)^{\frac{1}{3}}$.

3. *When* $r \in [1, 2]$, $R_{II}^N = r\frac{(2-r)^2}{2}$.

Lemma 27 1. *When* $r \in [0, \frac{1}{2}]$, $R_{II}^D = \int_r^1 (\frac{1}{3} + v_2) dv_2 + \int_0^r (\frac{1}{3} + v_2 + (r - v_2)^2(-\frac{2r}{3} - \frac{4v_2}{3})) dv_2$.

2. *When* $r \in (\frac{1}{2}, 1)$, $R_{II}^D = \int_r^1 (\frac{1}{3} + v_2) dv_2 + \int_{r-\frac{1}{2}}^r (\frac{1}{3} + v_2 + (r - v_2)^2(-\frac{2r}{3} - \frac{4v_2}{3})) dv_2 + \int_0^{r-\frac{1}{2}} r(1 - r + v_2) dv_2$.

3. *When* $r \in [1, \frac{3}{2})$, $R_{II}^D = \int_{r-\frac{1}{2}}^1 (\frac{1}{3} + v_2 + (r - v_2)^2(-\frac{2r}{3} - \frac{4v_2}{3})) dv_2 + \int_{r-1}^{r-\frac{1}{2}} r(1 - r + v_2) dv_2$.

4. *When* $r \in [\frac{3}{2}, 2]$, $R_{II}^D = \int_{r-1}^1 r(1 - r + v_2) dv_2$.

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