CPS 231 Exam 1

Fall 2003

1:00-2:25, Tuesday October 14th
Closed book exam

NAME:____

Problem	Max	Obtained
1	10	
2 (a)	10	
2 (b)	10	
3 (a)	10	
3 (b)	10	
4	15	
5 (a)	10	
5 (b)	10	
5 (c)	15	
Total	100	

Comments:

- You can use any of the algorithms covered in class without describing them.
- When describing an algorithm, remember to include an argument for both correctness and running time.

[*10 points*] **Problem 1:**

1. The summation $\sum_{i=0}^{\lg n} (\frac{1}{2})^i$ is $\Theta(\qquad)$.

2. is it true that $\sqrt{n} = O(2^{\log_2 n})$?

3. The best case running time of Quicksort is $\Theta(\qquad)$.

4. Given a heap with n elements, is it true that you can search for an element in $O(\log n)$ time?

5. Assume you have n positive integers in the range 1 through k. Counting Sort sorts the n integers in $O(\qquad)$ time using $O(\qquad)$ additional space.

[*20 points*] **Problem 2:**

a) Using the iteration method find an asymptotic tight bound for the recurrence:

$$T(n) = \begin{cases} 1 & \text{if } n \leq 3 \\ T(\sqrt{n}) + 1 & \text{if } n \geq 4 \end{cases}$$

b) Show using the substitution method (induction) that the recurrence above has solution $T(n) = O(\lg \lg n)$.

[*20 points*] **Problem 3:**

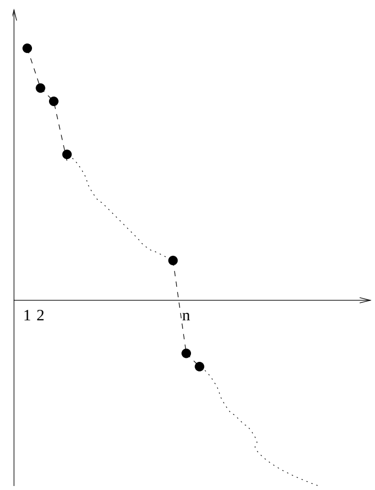
Let A be an array of n (not necessarily distinct) integers.

a) Describe an $O(n)$-algorithm to test whether any item occurs more than $\lceil n/2 \rceil$ times in A.

b) Describe an $O(n)$-algorithm to test whether any item occurs more than $\lceil n/4 \rceil$ times in A.

[*15 points*] **Problem 4:**

In this problem we consider a *monotonically decreasing* function $f : N \rightarrow Z$ (that is, a function defined on the natural numbers taking integer values, such that $f(i) > f(i+1)$). Assuming we can evaluate f at any i in constant time, we want to find $n = \min\{i \in N | f(i) \leq 0\}$ (that is, we want to find the value where f becomes negative).



We can obviously solve the problem in $O(n)$ time by evaluating $f(1), f(2), f(3), \ldots f(n)$. Describe an $O(\log n)$ algorithm.

(*Hint*: Evaluate f on $O(\log n)$ carefully chosen values between 1 and $2n$ - but remember that you do not know n initially).

[*35 points*] **Problem 5:**

The *maximum partial sum* problem (MPS) is defined as follows. Given an array $A[1..n]$ of integers, find values of i and j with $1 \le i \le j \le n$ such that

$$\sum_{k=i}^{j} A[k]$$

is maximized.

Example: For the array [4,-5,6,7,8,-10,5], the solution to MPS is $i = 3$ and $j = 5$ (sum 21).

To help us design an efficient algorithm for the maximum partial sum problem, we consider the *left position ℓ maximal partial sum* problem ($LMPS_\ell$). This problem consists of finding value j with $\ell \le j \le n$ such that

$$\sum_{k=\ell}^{j} A[k]$$

is maximized. Similarly, the *right position r maximal partial sum* problem ($RMPS_r$), consists of finding value i with $1 \le i \le r$ such that

$$\sum_{k=i}^{r} A[k]$$

is maximized.

Example: For the array [4,-5,6,7,8,-10,5] the solution to e.g. $LMPS_4$ is $j = 5$ (sum 15) and the solution to $RMPS_7$ is $i = 3$ (sum 16).

a) Describe $O(n)$ time algorithms for solving $LMPS_\ell$ and $RMPS_r$ for given ℓ and r.

b) Using an $O(n)$ time algorithm for $LMPS_\ell$, describe a simple $O(n^2)$ algorithm for solving MPS.

c) Using $O(n)$ time algorithms for $LMPS_\ell$ and $RMPS_r$, describe an $O(n \log n)$ divide-and-conquer algorithm for solving MPS.